Celestial Navigation
Practical Theory and Application of Principles

By Ron Davidson
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Preface

I grew up on the Jersey Shore very near the entrance to New York harbor and was fascinated by the comings and goings of the ships, passing the Ambrose and Scotland light ships that I would watch from my window at night. I wondered how these mariners could navigate these great ships from ports hundreds or thousands of miles distant and find the narrow entrance to New York harbor.

Celestial navigation was always shrouded in mystery that so intrigued me that I eventually began a journey of discovery. However, one of the most difficult tasks for me, after delving into the arcane knowledge presented in many reference books on the subject, was trying to articulate the “big picture” of how celestial navigation worked. Most writings were full of detailed cookbook instructions and mathematical formulas but frustratingly sparse on the overview of the critical scientific basis and principles of why and how celestial navigation works.

This guide represents my efforts at learning and teaching myself ‘celestial’ to present the big picture of how celestial principles work without too many magical formulas. I then cover the procedures of Celestial Sight Reduction with examples for Sun, Moon, planet, and star sight reductions.


My intention is for this book to be used as a self-teaching tool for those who have the desire to learn celestial from the natural, academic, and practical points of view. We will use our celestial navigation knowledge and the Law of Cosines formulas to solve sextant sights for position.

With the prevalence of computers, tablets, and handheld electronic calculators, the traditional methods of using sight-reduction tables with pre-computed solutions will scarcely be mentioned here. I am referring to the typical methods using Pub 249 SIGHT REDUCTION TABLES FOR AIR NAVIGATION and Pub 229 SIGHT REDUCTION TABLES FOR MARINE NAVIGATION. Rather, the essential background and equations to the solutions will be presented such that the reader can calculate the answers precisely with a hand calculator and understand the why they work. You will need a scientific calculator, those having trigonometric functions and their inverse functions. To those readers familiar with ‘celestial’, they will notice that I have departed from the usual standards found in celestial navigation texts.

Ron Davidson

Caveat Emptor!

This book is for educational purposes only. Any person using the information within these pages does so entirely at their own risk. I assume no liabilities of any form from any party:
The Essence of Celestial Navigation

You are standing somewhere on the Earth’s surface and you’re not exactly sure where that is latitude and longitude-wise, but you have some idea. We call that location your deduced reckoning (DR) position.

Now imagine that the sun’s light were focused like a laser pointer shining directly down onto the Earth’s surface and where it hits the surface it is marked with an X. We call that spot the sun’s geographic position (GP).

We now have the Earth with two spots on its surface: our DR position and an X marking the GP of the sun. In celestial navigation, we measure, and plot the distance between these two spots. If we knew the distance to the sun’s GP at a particular moment, then we could draw a circle on the Earth’s surface with a radius equal to that distance (a Circle of Position (COP)), and we would surmise we were somewhere on that circle of position.

Using a sextant it is easy to measure the distance between the two locations. The distance between our position and the sun’s GP is directly related to the altitude of the sun as measured by the sextant. The higher the altitude, the closer you are to the GP. If the sun were directly overhead, you would be at the GP and your circle of position would be quite small. If the sun were near the horizon, you would be thousands of miles from the GP and the circle of position would be very large. Either way, your position would be somewhere on the circle of position.

To determine the distance between our position and the GP, we subtract the measured sextant altitude from 90° to determine Co-Altitude and then multiply the Co-Altitude by 60. The result is our distance from the GP in nautical miles. For example, if the measured sextant altitude were 61°, as might be measured near midday in summer from Puget Sound, Washington you would be (90 – 61 = 29 X 60) 1,740 miles from the GP. OK, but are we at our DR position?

At sea, we have no visual clues, such as buoys, points of land, etc. to help us verify our DR. Our DR is all we have. We have measured the altitude from our present position, so we ask, what would be the altitude if measured from our DR position? If we knew the altitude from our DR, we could compare our measured altitude to the altitude from the DR to see if they are the same or differ. If the altitudes are the same, we must have been at our DR when we measured the altitude. If the altitudes differ, we must have been somewhere other than our DR position. Using the latitude and longitude of our DR along with data we extract from the Nautical Almanac, we can calculate what the altitude of the celestial body would be if measured from our DR position. That calculating process is called Sight Reduction and will be covered later. We now have two altitudes, our measured altitude and the altitude we calculated and can now compare them to learn if we were at our DR when we measured the altitude and whether we are closer to or farther from the GP of the body.
Additionally, if we sighted a second celestial body, say the Moon, using the Moon’s GP we would have two GPs. And if we measured the altitude of the Moon with our sextant, we would have two circles of position on which we were located. Basic navigation theory tells us that if we are located on two different circles of position, we must be located at one of the two places where the circles intersect; the one that is closest to our DR position.

We have now determined our position on the Earth’s surface. There are many more details that we need to take into account however. We must apply corrections to our sextant reading necessary to account for the fact that our eyes are not at sea level and for the refraction (bending) of light by the atmosphere we experience when viewing celestial bodies. We also need to learn about the Navigational Triangle that allows us to associate measured altitude to distance to the GP. And lastly, our Circles of Position are very large so, how do we plot them? These details are covered in more detail later.

The Overview

This is a general overview of the celestial process don’t worry if you don’t understand every detail. In preparation to taking a sextant sighting we first determine, record, and plot our deduced reckoning (DR) position. We then use our sextant to measure the altitude of our selected celestial body above the visible horizon and record the altitude measured (Hs) along with the exact time (second, minute, and hour) of our sighting. Once that is completed we next apply some corrections (covered later) to our measurement to arrive at our Observed Altitude (Ho). The altitude measured tells us (indirectly (explained below)) our location’s distance from the GP of the selected celestial body.

We now must ask: Were we actually located at our DR position when we took the sighting? To what can we compare our measurement? How can we verify our location? Here’s how: The nature of the data contained in the Nautical Almanac is detailed such that we can use the latitude and longitude of our DR position to calculate what the altitude of the sighted celestial body would be if measured from that latitude and longitude at the time we took our sighting! Once the altitude calculation (Hc) is completed we can then compare the altitude we calculated (Hc) to the altitude we actually measured (Ho).

If the two altitudes are identical then our location is confirmed to be at our DR position. If the two altitudes differ then our location is not at our DR. Then where are we located relative to the GP? The answer is simple: What is the difference between our two altitudes Hc & Ho? This difference is called the intercept. We learned previously that one minute of angle is equal to one nautical mile. So, for example, if our Hc were say 31° 41.8’ and our Ho was 31° 38.9 the difference between Hc & Ho is 2.9’ or 2.9 nautical miles. This tells us that we are located 2.9 nautical miles from our DR position, but in which direction? Are we closer to the celestial GP or farther away? Once again the answer is pretty simple. If Hc is greater than Ho we must be farther away from the GP because the altitude we measured (Ho) is smaller than calculated (Hc). If Ho were greater than Hc we must be closer to (toward) the GP because the altitude we measured (Ho) is greater than calculated (Hc).

In order to plot our position accurately, we also need an accurate bearing (azimuth) to the GP. Where can we find one? Once again we can use the data from the Nautical Almanac to calculate the azimuth from our location to the Geographic Position (GP) of the selected celestial body that we must have been on at the moment we took our sextant measurement.

Once we have calculated the azimuth, we lightly plot the azimuth through our DR position and mark our intercept (2.9 nautical miles in this example) on that azimuth in the direction opposite the GP
(AWAY). Again, it is plotted away because Hc is greater than Ho in this example. Had Ho been greater than Hc we would plot the intercept TOWARD the GP.

**Rule: If Ho > Hc - Plot the intercept in the direction of (TOWARD) the GP; If Ho < Hc - Plot the intercept farther AWAY from the GP.**

The point plotted is our estimated position (EP). It is an EP because it is based upon a single sextant sighting. If we solved a sighting on a second celestial body (within 20 minutes of time) we could then plot both points for a "fix" of our position.

**Altitudes and Co-Altitudes**

**Background:** Those interested in celestial navigation understand that knowledge of working with angles (degrees, minutes, and tenths of minutes) is required and many find that alarming or intimidating. Yes, angles are involved and spherical trigonometry is ultimately employed to obtain the results, however, navigators do not need to study the theory behind spherical trigonometry, they just need to know some basic arithmetic and how to use the two formulas provided. It is easier to understand the celestial navigation process if we first understand a few basic concepts being employed.

**Concept #1:** Angles and Complements. The figure below shows a 90° angle between the vertical and horizontal lines and also shows an angle of 30° from horizontal and asks for the complement of the angle. The complement of an angle is the difference between the angle shown and 90° i.e. 90° - the angle, in this case, 90° - 30° = 60°.

**ANGLES AND COMPLEMENTS**

What is the complement of 30°?

90° - 30° = 60°

This next figure shows another example of finding the complement of an angle.

**ANGLES AND COMPLEMENTS**

What is the complement of 60°?

90° - 60° = 30°
Here are two more examples although these examples are using a person’s location and Earth’s latitude as the angle the concept, however, remains the same. You should have little trouble identifying the complements.

**Concept #2**: Distances: Mathematicians have determined that 1° of latitude equals 60 nautical miles (nm) and 01’ is 1 nm. Based on that knowledge and knowledge of complements, we can calculate the distance between a person’s location and another known point. For instance, if two persons were separated by 3° we could calculate the distance between them by 3° X 60nm per degree = 180 nautical miles distance.

Based on the two figures above we could then calculate our distance from the North Pole (Pn):

If we were located at 15° N Latitude, how far are we from the Pn?

\[90° - 15° = 75° = 75 \times 60 = 4500 \text{ nm}.\]

If we were located at 60° N Latitude, how far are we from the Pn?

\[90° - 60° = 30° = 30 \times 60 = 1800 \text{ nm}.\]

The complement of our latitude tells us the distance to the North Pole! The complement of latitude is called Co-Latitude.

**Concept #3**: Circle of Position. The distance between our location and some known point creates a “Circle of Position” (COP). The figure below shows a vessel that has detected via radar, a buoy at a range of 5nm. If the navigator located that buoy on a chart and, using a drawing compass set to the 5nm distance, the navigator could place the point of the compass on the buoy and draw a circle around that buoy creating a circle of position with a radius of 5nm. The navigator would know that the vessel is located somewhere on that circle. To determine where on the circle, the navigator would read the radar bearing to the buoy and plot that bearing on the chart. Where the bearing and the COP intersect would be the vessel’s location.
Concept #4: Measuring altitude using a sextant. A navigator observes a celestial body in the sky and uses a sextant to measure the altitude of the body above the horizon. The horizon constitutes the horizontal reference line and the measured angle provides, indirectly, the complement angle as shown in the figure below:

For celestial navigation, the COP concept #3 is employed. The celestial body’s Geographical Position (GP) is the “buoy” and the 3,480nm is the radius of the circle of position from the GP of the celestial body. The mariner is located somewhere on that COP. Plotting that large circle on a chart as we did the radar COP, is impractical because the circle’s radius is so huge! Even if we had a chart of small enough scale to plot the circle, the scale of the chart would make it impossible to plot an accurate position. The solution to this dilemma is a mathematical one versus a mechanical one and will be covered in more detail later.

The distance of 3,480nm is the distance between the mariner’s location and the location of the GP. An imaginary line connecting the celestial body to the Earth’s center passes through the Earth’s surface at the GP. The navigator determines the precise location (latitude and longitude) of the GP from the data recorded in the Nautical Almanac. The complement of the measured altitude measures the distance between the mariner’s location and the location of the GP. In the figure above that distance is 3,480nm. The actual sextant measurement (with some corrections applied) of a body’s altitude is called the “Observed Altitude” and is abbreviated Ho; H for height or altitude and o for observed. Once a sextant measurement of the altitude of a celestial body is made and an Observed Altitude (Ho) determined, the Co-Altitude (90° - Ho) determines the radius of the COP from the mariner’s position to the body’s GP.

Concept #5: The Calculated Altitude. To determine additional position information, the mariner needs a “known value” of altitude to compare to the “observed altitude”. To achieve this the mariner selects a “reference position” (a latitude and longitude) in the vicinity of the deduced reckoning (DR)
position. Mariners typically use the DR position however any position within 30nm of the DR will work just as well. Many celestial navigation texts refer to this reference position as an “Assumed Position” (AP) but that is a misnomer. The mariner is not assuming he/she is located there; it is just a position chosen as a reference point.

Many students get confused at this point because they view the DR position as nebulous and uncertain. That is not true; the DR is a specific point of latitude and longitude that can be accurately plotted. What is uncertain is the mariner’s location; he/she may be located at the DR or may not. The mariner’s location is uncertain and is what must be determined.

Using the reference position chosen and data from the almanac related to the celestial body observed the latitude and longitude of the GP of the celestial body can be determined. The altitude of the celestial body that would be measured from the reference position can then be calculated. The altitude calculated is labelled Hc. The altitude calculated provides, indirectly, the complement (Co-Altitude) of the angle and therefore, provides the distance between the GP and the reference position.

There are now two distances to the GP: 1) the distance based upon the measured Observed Altitude (Ho) and 2) the distance based upon the Calculated Altitude (Hc) from the reference position. A comparison of Ho to Hc is then made to find the difference. The difference between Ho and Hc is called the Intercept and is the distance the mariner’s location was offset from the reference position at the moment the mariner measured the observed altitude.

For example, if the mariner measured a Ho of 32° 28.0’ and then calculated an altitude (Hc) of 32° 26.5’ from a reference position, the difference of 01.5’ tells the mariner that his/her location was offset 01.5nm from the reference position at the time they measured Ho.

\[
\begin{align*}
\text{The Co-Altitude of Ho is } & 90° - 32° 28.0’ = 57° 32.0’ = 3452\text{nm from the GP.} \\
\text{The Co-Altitude of Hc is } & 90° - 32° 26.5’ = 57° 33.5’ = 3453.5\text{nm from the GP.}
\end{align*}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Ho} & 32° 28.0’ & \text{Co-Alt} & 57° 32.0’ & \text{Distance from GP (radius of COP)} & 3452.0\text{nm} \\
\hline
\text{Hc} & 32° 26.5’ & \text{Co-Alt} & 57° 33.5’ & \text{Distance from GP (radius of COP)} & 3453.5\text{nm} \\
\hline
\text{Diff} & 01.5’ & 01.5’ & & 01.5\text{nm} \\
\hline
\end{array}
\]

Notice the same answer of 01.5 is achieved whether we compare Ho to Hc, Co-Altitude to Co-Altitude, or Co-Altitude distance to Co-Altitude distance. No matter which calculation is chosen, the result is 01.5nm. Notice also that we arrive at the 01.5nm difference just by comparing Ho to Hc, so we can determine the Intercept by that method alone and ignore the other calculations. This is the mathematical solution that eliminates the need to plot huge Circles of Position. We have determined that had we drawn the two huge COPs they would have created two arcs in our vicinity with 01.5nm separation between them. This phenomenon was first discovered by the French navigator Marcq de St. Hilaire in the 1870’s and has become known as the Altitude-Intercept method of sight reduction.

Also note that, in this example, Ho is greater than Hc and that the Co-Ho distance from the GP is less than the Co-Hc distance from GP. If Ho is greater than Hc the mariner’s position must be plotted closer to (toward) the GP from the reference position. If Ho were less than Hc the mariner’s position must be plotted farther away from the GP than the reference position.

How do we know if we need to plot TOWARD the GP or AWAY from the GP? Find a ceiling light or fan in your home. Stand near a wall and point at the light or fan. Step toward the light or fan while
continuing to point. See how you have to raise your arm to a greater angle as you near the light or fan? The nearer you are to the spot directly below the light or fan the greater the angle. So, if $Ho$ is greater than $Hc$ we must have been closer (toward) to the GP at the time of the sight than the reference position. If $Ho < Hc$ we must have been farther away. It’s that simple.

The following figure illustrates an example where $Ho > Hc$. AP is the reference position chosen, the Intercept is the difference in nautical miles, between $Ho$ & $Hc$, and LOP is the resultant Line of Position that is plotted at 90° to the azimuth to the GP. The LOP is the line the mariner must have been on at the time of the sight and is actually a small segment of the COP from the GP. It can be drawn as a straight line because the circle is so large. The ship’s position at the time of the sight is where the azimuth to the GP and the segment of the $Ho$ COP intersect. The vessel’s estimated position is where the LOP and azimuth intersect.

![Diagram 1](image1)

The next figure illustrates an example where $Ho < Hc$. Again, the AP is the reference position chosen, the Intercept is the difference in nautical miles between $Ho$ & $Hc$, and LOP is the resultant Line of Position that is plotted at 90° to the azimuth to the GP. The LOP is the line the mariner must have been on at the time of the sighting. The ship’s position at the time of the sight is where the azimuth to the GP and the segment of the $Ho$ COP intersect. As can be readily seen in the figure, the vessel’s position is “Away” from the AP (GP) by the Intercept distance.

![Diagram 2](image2)

This important observation creates a plotting rule:

$Ho > Hc = “Toward”$ the GP; $Ho < Hc = “Away”$ from the GP.
Concept #6: The Navigational Triangle. The narrative above has, I hope, raised a significant question: “How did we arrive at a calculated altitude (Hc) used to achieve the resultant Intercept?” The answer is, by solving the Navigational Triangle, commonly referred to as the process of Sight Reduction. The Navigational Triangle is a large spherical triangle and the Law of Cosines formulas we’ll use to solve it will ensure an accurate solution.

The three vertices of the Navigational Triangle are: 1) The reference (AP or DR) position chosen. 2) The “elevated” pole; the pole of the Earth nearest the mariner’s location. 3) The Geographical Position (GP) of the celestial body, determined from Nautical Almanac data extracted based on the body observed. The figure below shows the Navigational Triangle in action. Study it closely.

The sides of the triangle are Co-L (90 – latitude of the reference position); Co-Dec (90 – declination of the celestial body). Declination is the term used in the almanac to describe a celestial body’s latitude above or below the celestial equator and is recorded in the almanac, and Co-H the Co-Altitude of the altitude we’ll calculate (Hc). Co-L is an easily determined value. Co-Dec is also an easily determined value (derived from almanac data). Co-H is the unknown side and the value for which we are solving.

The longitude of the geographical position is also known and the “Hour Circle of Body” (derived from the almanac data) provides the longitude of the body’s GP. The difference in longitude between the DR and the GP is called the Local Hour Angle (LHA), always measured westerly, and is also easily calculated by comparing the two longitudes. We now have a triangle with two known sides and an angle between two sides and that is enough information to solve for the unknown side.

We’ll also solve for angle Z and, from that angle, we can deduce angle Zn. Zn is the true azimuth (measured from True North) from the reference position to the GP of the body. Look closely at the figure above and you’ll see that, in this case, Zn = 360° - Z.

We mathematically solve the navigational triangle using the Law of Cosines formulas below:

\[
\sin Hc = (\cos LHA \times \cos Lat \times \cos Dec) + (\sin Lat \times \sin Dec)
\]

\[
\cos Z = (\sin Dec - (\sin Lat \times \sin Hc)) / (\cos Lat \times \cos Hc)
\]

Although these formulas may look intimidating, using them is straight-forward; we just place the appropriate values into the formulas using a scientific calculator and let it do the work.
When using these formulas we must follow another rule:

*Latitude (North or South) is always entered as a positive value; declination is entered as positive if it has the same name as latitude (both North or both South), negative otherwise.*

The results of the calculations provide the calculated altitude (Hc) which is then compared to Ho to determine the Intercept and angle Z which is used to determine the true azimuth (Zn) from the DR to the celestial body’s GP. We can then plot the reference position on a Constant Latitude Scale Small Area Plotting Sheet (CLS SAPS) and plot the intercept along azimuth Zn either “Toward” the GP or “Away” from the GP as indicated by the rule Ho > Hc = Toward or Ho < Hc = Away. The point plotted is an Estimated Position (EP). It is an EP because it is the result of only a single sight. If we took a sight on a second celestial body within 20 minutes of the first sight then solved and plotted the second sight we could establish a “fix” of our position; provided that the longitudinal separation of the bodies were large enough to create a “good” crossing angle of the resulting Lines of Position (LOPs).

**The Concepts at Work**

Because the Earth is rotating at 900 knots (15° per hour X 60nm per degree = 900), the recording of the precise time of observation is vitally important and allows the appropriate nautical almanac data to be extracted for the precise time recorded.

The light rays from the celestial bodies travel to the Earth essentially, as parallel rays as shown in the figure at right. The task of the celestial navigator is to determine the angle between the light rays and vertical at the navigator’s location (angle ø in the figure). For a mariner aboard a moving vessel with no vertical reference, this angle proves too difficult to measure. To overcome this dilemma, the navigator instead measures the altitude angle from the horizon to the body and then by subtracting the angle measured from 90° determines the Co-Altitude, which is the angle of interest (ø). Learning the Co-Altitude provides the mariner with the distance between the navigator’s location and the celestial body’s GP, creating a huge Circle of Position which is then compared to a “reference sight” distance, once the reference sight is reduced by extracting appropriate data from the nautical almanac and, after some arithmetic is performed, the navigator’s position is determined from the result.

**A Bit of History**

When we think of celestial navigation, for many, our thoughts drift to the age of exploration and names like Magellan, da Gama, Vespucci, Columbus, Drake, Hudson, Cook, (circa 1454 -1779) however, the principles used in celestial navigation were discovered well before that time.

The Greek astronomer and mathematician Eratosthenes (276 - 194 BC) made some practical observations that lead to the discovery of the principles used today in celestial navigation.

Eratosthenes observed that at noon, around the time of the summer solstice, vertical posts at Alexandria cast a shadow on the ground, whereas at Syene (present day Aswan) it was reported that posts there cast no shadow at that time and the sun illuminated the entire bottom of a well at noontime.
This observation led Eratosthenes to believe the earth must be spherical and the sun’s rays are essentially parallel to each other. This inference enabled him to make some calculations that were truly elegant in their simplicity and that proved the earth is spherical and moreover, allowed him to calculate the earth’s circumference at 25,000 miles (today’s measurement is 24,901 miles). He determined the sun's rays were vertical at Syene and 7¼° from vertical at Alexandria or 1/50th of a circle. He then measured the distance between Alexandria and Syene at 500 miles, to calculate 50 * 500 = 25,000.

Although Eratosthenes made some assumptions that affected the accuracy of his measurements, many of today’s experts are astonished at the accuracy of his calculations. So, how are Eratosthenes’ observations related to celestial navigation? They provided a method to calculate the distance (see d in Figure 1.1 above) between two places on the earth using Eratosthenes’ angle ø! We use a derivative of Eratosthenes’ formula in celestial navigation.

**The Mariner’s Angle**
A ship’s motion at sea makes measuring Eratosthenes’ angle from vertical too difficult to measure. Instead, navigators use a sextant to measure the angle from the horizontal, as seen in Figure 1.2 below. The sextant measures the sun’s altitude above the visible horizon and we find that the altitude is 90° minus the angle of Eratosthenes! In celestial navigation we call Eratosthenes’ angle the Co-Altitude. The two angles are complements of each other, meaning their sum is 90°. From his observations comes the formula distance = 60 X ø. This formula has become the guts of celestial navigation.
**The Equal-Altitude Line of Position (Circle of Position)**

Figure 1.3 below shows us a graphic of the sun’s rays in relationship to the spherical earth. The altitude and co-altitude of the sun’s rays at one observer’s position are shown at the top of the figure. All the other observers shown in the figure are located where they see the identical altitude. We can see that these equal-altitude locations must lie on a circle centered on the sun’s geographical position (GP) with a radius equal to the observer-to-GP distance. This radius is the same distance as $d$ in the example of Eratosthenes’ as shown in Figure 1.1, the radius length is just 60 nm (60 nm per degree) times the co-altitude. So **distance to GP = 60 nm x Co-altitude**.

So, by measuring the sun’s altitude and subtracting that altitude from 90°, we learn that our position lies somewhere on this circle of equal altitude. For example, if the altitude we measured was 21° 23.7’ then $90 - 21° 23.7' = 68° 36.3'$ (68.605°). Now, using Eratosthenes’ formula $d = 60 \times 68.605 = 4116.3$ nm is the radius of the circle of equal altitude. We are located somewhere on that circle. Our job becomes one of narrowing the possibilities to find a plausible location.

The altitude measured by each observer depends on his/her distance from the sun’s GP. The closer you, as the observer, are to the sun’s GP, the greater the observed altitude, and conversely, the
farther away the observer is, the less the altitude. If you were located at the sun’s GP, the sun would be directly overhead, its altitude would be 90°, and your co-altitude would be zero thus your distance from the GP would be zero nm (90° - 90° = 0). To see what I mean, find a room in your house with a ceiling light. Position yourself near a wall and point at the light. Now step toward the light while continuing to point at the light. See how you have to raise your arm as you move closer? The altitude increases as you get closer.

Since one sextant observation just tells us we’re somewhere on this large circle of position, we need more information to fix our location on this circle. With just one observation and using celestial mathematics we’ll be able to identify our Estimated Position (EP) with just a few miles of error. In celestial navigation, to fix our position is done by making an observation of a second celestial body to obtain a second circle of position. With two observations, we’ll be able to develop a “fix” of our position as shown in figure 1507 below.

As shown in Figure 1507 above, these two circles of position would intersect in two places, leaving an ambiguity between the two intersections as our position. However, as we can also see in the figure, these circles of position are quite large, making the elimination of one of the two intersections quite easy as one intersection is ENE of Cuba and the other is East of Argentina. Two different hemispheres!

Using the Nautical Almanac
The British first published the Nautical Almanac and Astronomical Ephemeris in 1766, with data for 1767. The Nautical Almanac contains data that we can use to determine the precise Geographical Position (GP) of the celestial bodies used in navigation (Sun, Moon, Venus, Mars, Jupiter, Saturn, and 57 selected stars) at any second of time throughout the year of the almanac. By knowing this GP location and our observed altitude taken with the sextant, we learn the radius and location of our circle of position. Remember the GP is at the center.
The celestial data contained in the almanac is tabulated for each whole hour of each day in Universal Coordinated Time abbreviated UT. This fact makes it incumbent on the navigator to possess the knowledge and ability to convert local ship’s time to UT before he/she can extract appropriate data from the almanac. There will be a more detailed discussion of time later.

The Limitations of Mechanical Methods
Plotting such huge circles of position on our charts however, is impractical for two reasons: 1) a chart covering an area that large would have such a small scale that accurate plotting of our position would not be possible and conversely, 2) a chart with a large enough scale to allow accurate plotting would be physically too large and impractical for use on the vessel.

Since mechanical methods will not work, we’ll have to use a mathematical solution. We will not delve into how the mathematics, known for over one thousand years, were developed we’ll just use it.

The Sextant
A sextant is a very sophisticated instrument for measuring angles. Most of us are familiar with plastic protractors used for measuring angles in high school geometry or trig. These have one division for each degree, and most of us would agree that the one degree divisions are pretty small. Now consider taking each of those degrees and dividing it into 60 equal parts. Each of these parts we call one minute. A marine sextant can easily measure angles down to the minute.

If we then divide each of these minutes into 60 parts, we have created 3600 arc seconds in only one degree. That is pretty fine division!

A marine sextant subdivides minutes, but not quite as finely as one second. Instead, it takes each minute of arc and divides it into fifths (but marked in tenths) as 0.0’, 0.2’, 0.4’, 0.6’, and 0.8’
Reading the Sextant Scales
Most sextants have three scales that give readings to 2/10\textsuperscript{th} of a minute. The scale on the frame is called the “arc”; each division of the arc equals one degree.

**To read the number of degrees:**
Find the lines on the arc that are closest to the index line on the index arm. The index line is usually somewhere between two lines. The correct reading is usually that of the lower value, i.e., the line to the right of the index line.

**To read fractions of a degree:**
Use the two scales involving the micrometer drum at the side of the index arm. The outer revolving drum scale indicates minutes of arc (one minute equals 1/60 of a degree), while the stationary vernier reads to 2/10\textsuperscript{th} of a minute.

**To read the number of minutes:**
Find the single LONG line at the top of the vernier. The line on the drum scale that is opposite this line gives the number of minutes. If the line on the vernier is between two lines on the drum, choose the line of lower value.

**To read fractions of a minute:**
Find the SHORT line of the vernier that is opposite to a line on the drum. Count the number of spaces this line is away from the long line at the top of the vernier. Each one equals 2/10\textsuperscript{th} of a minute.

To adjust the sextant to find index error: (Index mirror is not perpendicular to the frame)
Set the instrument at 0° 00’ and look at the horizon. Keeping the sextant close to your eye, turn the micrometer until the two horizon images move exactly together. Read and record the scales. The reading is the Index Error. If the reading is above zero, (on the arc) the sextant reads high and that amount must be subtracted any sextant measurement. If the reading is below zero (off the arc (60 – reading)), the sextant reads low and that amount must be added to any sextant measurement. This should be done 2 or 3 times to confirm.
Using the Sextant

1. Prior to a sight taking session, check and record the Index Error as explained above.

2. Most sextants include a lanyard that you should lay around your neck to prevent the sextant from falling overboard or damage if accidentally dropped.

3. When taking sextant sightings of the sun or a bright moon, USE THE SUN SHADES! Start by applying all shades then remove the shades, lightest first, until you can comfortably view the sun yet discern the natural horizon. Choose a combination of shades that gives you a clear image without glare. The sun (or moon) should appear as a crisp orange disk.

4. Have an accurate watch and a small notepad on-hand to record your times and sextant readings. Until you build your confidence using the sextant, it is recommended you have an assistant do the recording.

5. To record the accurate time of a sight, once you have the sextant and celestial body properly aligned, say “Mark!” the assistant then reads the time seconds first then minutes and hour. Then read and record the sextant reading.

Measuring the Sun’s Altitude:

1. Use index shades to protect your eyes, as discussed above.

2. Use the horizon shades to darken the clear section of the horizon mirror so that it acts as a semi-mirror. The horizon will still be visible through it, but the sun’s image will be reflected.

3. Stand facing the sun with the sextant in your right hand.

4. With your left hand on the quick release levers of the index arm, look through the eyepiece at the horizon and move the index arm until the sun is visible through the two mirrors and index shades.

5. Release the levers and, while slowly rocking the entire sextant from side to side, use the fine adjustment drum to bring the sun’s image down to just touch the horizon with its lower edge (lower limb). The sun’s image should travel a short arc that is made to touch the horizon.
6. Call "Mark!" for your assistant to record the time then read and record the sextant scales.

**The Captain Marq de St Hilaire Method (Intercept & Azimuth)**

The purpose of sight reduction is to determine the latitude and longitude of some point on the all-important circular equal-altitude COP and to do it in a relatively simple manner. After all, mariners should not have to be mathematicians in order to navigate. Captain St Hilaire published his method in 1875 and it meets those requirements.

Captain St Hilaire discovered a method of reducing a celestial observation for finding position using the circle of equal altitudes that does NOT require attempting to plot these huge circles on our charts.

Angles of celestial bodies above the horizon, measured using a sextant, are termed "altitudes" and the difference between two altitudes, once converted to nautical miles, is termed the "intercept". We'll see how the entire process is accomplished as we continue.

The **altitude-intercept** method involves observations with a sextant, but there is more work to be done since a sextant cannot, by itself, "yield your position." In this method, we must have data from both the sextant and also a very accurate watch. (Today’s quartz and digital watches are very accurate).

Taking a sextant observation is entirely worthless unless we know precisely when that observation was made. And even then, at least two such observations on two different objects are required to yield your position, known as a **fix**. The best a single observation can yield is an “**estimated position**”.

**Do altitudes tell us our position?**

Not directly, but they are a critical part of the whole process. The altitude at which a body appears in the sky is related to three conditions:

- The location of the body in space.
- The exact time of the observation.
- The position of the observer on the Earth.

Do you now get a hint of where this is going? This is like a high school algebra problem where you are given enough information to allow you to solve for that one bit you don't know. From the viewpoint of the navigator, here is where you get the data you need:

- The location of celestial bodies in space is compiled into the Nautical Almanac.
- The time is measured. “Old salts” used a chronometer, but today the typical digital quartz watch is more accurate than the very best chronometers of only two generations ago.
- The altitude of a star, a planet, the Moon or the Sun is measured using a sextant.

Now 3 things of 4 are known, and the final item of interest can be solved for; namely, the position of the observer on the Earth. That is the theory of celestial navigation.
What is an intercept?

It is going to be a lengthy explanation, so bear with me...It will be worth the effort.

To begin, first notice that the spot directly over your head is always directly over your head. That spot is termed your Zenith.

As strange as it may seem this is one statement that is actually very important to your understanding of how the altitude-intercept method works.

Please also imagine that you are standing on a flat, level surface. If you now stand one arm of a carpenter’s square between your feet pointing out toward the horizon, the other arm will point at that spot directly over your head, your zenith.

Now if to go just one step further... Imagine that the surface you’re standing on goes out as far as the eye can see. In other words, the surface extends all the way to your horizon.

If you are following this line of thought, it should be very clear now that the spot directly over your head is precisely 90° from the horizon the carpenter’s square is telling you so. And since the spot directly over your head is always directly over your head, it surely must be 90° from the horizon no matter where on Earth you may be standing.

Let’s pick out a very special zenith. Suppose that you are standing at the bottom of a lighthouse. Now there is a very bright light directly over your head that can be seen for miles around. I could ask you, "What is the altitude of the lighthouse light?" Now, knowing what you do about a zenith and an altitude, you would answer me without even a measurement: "Why 90°, of course; since the lighthouse light is directly over my head at my zenith". And your answer would be exactly correct.

Now comes a truly interesting and crucial point in navigation. Your answer is perfectly sound, but unless I happen to be standing right with you there in the lighthouse, I will disagree with you as to the altitude of the light.

How so?

Because the spot directly over my head is always directly over my head! And it is a different spot than yours. Since my zenith is still a zenith, it is precisely 90° from my horizon. Therefore, if I am not standing in the lighthouse with you, I will say that the altitude of the lighthouse light is less than 90°. If you doubt me, stand directly under a light fixture in your house. You will see that the angle between the floor and the light (your zenith) is 90°. Now take 3 steps backward and look at the light again. You will see that the angle between the floor, through you to the light is now less than 90°.

Fortunately for celestial navigation, there is a mathematical relationship that will tell me just what the altitude of the light will be. It depends on how far I am from the lighthouse and how tall the lighthouse is. See the diagram at right.

In this diagram, you are in the lighthouse and I’m somewhere outside.
The lighthouse is X feet tall.

I am standing some D feet away from the lighthouse.

The altitude of the light is 90° for you, and some altitude, H, for me.

Now going back to the right triangles of high school trigonometry, we remember that the tangent of an angle is equal to the length of the "opposite side" divided by the length of the "adjacent side", written

\[ \tan H = \left( \frac{X}{D} \right) \]

Of course, the situation envisioned is that I have my handy sextant to measure altitude H, I just so happen to know the height of the lighthouse H, and I can then whip out my calculator to solve for distance between you and me, namely D.

The intercept comes in because of two complications, one based in theory and one based in technology.

The previous diagram shows a "side view" of the lighthouse problem. The theoretical trouble is just this: With the information at hand, I cannot tell which "side" I am looking at. That is to say, I can tell the distance I may be from you, but not the direction. Am I D feet North of the lighthouse? D feet South? East? West? or somewhere in between. Here's an "overhead view" of all the possibilities.

In this diagram, we still see you in the lighthouse and me somewhere outside.

The Four Cardinal Directions around the horizon are shown.

I am standing some D feet away from the lighthouse.

Everywhere I may stand along the circle circumference the sextant will yield the same altitude of the light.

This very important phenomenon is called the Circle of Equal Altitude.

It also brings up something crucially important for developing the idea of an intercept.

Consider this point: if I were to stand off of the circle of equal altitude, my sextant reading would change. If I stand closer to you, the altitude will increase. It must because my zenith is getting closer to your zenith if I move inside the circle. If I take a step or two back, the altitude must decrease because my zenith is moving farther away.

Walking along a circle of equal altitude is something else you can try in your own room with that ceiling-mounted light again, and I strongly encouraged you to do so to affirm the point.

If we need direction, why don't we just use a compass and measure our bearing to the lighthouse? Then we would know everything we need.
Sounds good! And it is routinely done in coastal navigation, where angular precision is less important because you are sighting a stationary landmark relatively nearby.

When our "lighthouses" become celestial bodies, we run into a technological problem.

A sextant easily measures down to fractions of a minute. Well, a compass doesn't. No one has yet devised a compass that gives better than fractions of a degree, so compass measurements are over 60 times less accurate than sextant measurements. This means that their relative uncertainty is large to begin with. Additionally, in normal practice the distance to our celestial "lighthouse" will be thousands of miles, so we would be taking a large uncertainty and multiplying it by a very large number, making it even more uncertain.

As if that weren't enough, when it comes time to plot our fix, we have no drafting instruments that will divide angles more finely than perhaps half a degree. All these large uncertainties add up quickly, and even though we don't seek "scientific precision" in our work, this level of uncertainty is just unacceptable.

This explains why we must measure at least two altitudes on two different bodies for a fix. A two-body celestial fix is the intersection of two circles of equal altitude. If you could draw the circles out completely on a globe, they would resemble the MasterCard symbol, as shown:

I say only resemble because in the circles won't be of equal size, but the graphic makes the point.

Our position is where the circles intersect. However, two circles taken from celestial sights normally cross each other twice, but the intersect positions are hundreds or thousands of miles apart, so it is obvious which one is the position of our vessel. Since each altitude is known quite precisely, the intersection fix is far more precise than trying to take an altitude and a bearing, or azimuth, as it is called, of a single body.

The trouble appears when we go to actually plot our fix on the plotting sheet. The circles are huge and our plotting sheet too small. So, we approximate a small portion of the circle of equal altitude by a straight line, called a "line of position" as shown in the figure above and earlier in Figure 1507. This is more practical than trying to draw a huge circle, but we still must know where the center of the circle lies. That is to say, we need to know what the azimuth (bearing) to our "lighthouse" would be if we could measure it.

The final answer lies in some very real complications to the lighthouse pictures. If you haven't guessed by now, the lighthouses of which we are speaking are the Sun, Moon, planets, and stars. They have a huge advantage over man-made lighthouses in that they can be seen over entire hemispheres rather than just the local coastal region. But using celestial bodies leads to certain complications:

As viewed from the Earth, celestial bodies are moving in a very complex manner. With a few exceptions, stars rise, climb higher in the sky for a few hours, then descend toward the horizon and...
set. They appear again the next evening, but about 1° West of where they were the previous night at the same time. After a season or two, the whole sky looks different as once-familiar stars get lost in the glare of day for a few months, only to re-appear after a year has passed, right back where we first noticed them. This means that even in those extraordinarily rare instances that a bright star is in our zenith, it won’t stay there long.

We also can’t tell how “tall” these “lighthouses” are. Remember that our trigonometry solution depended on knowing the height of the lighthouse.

The Earth is spherical, not flat; and the sky also appears to be a big round dome over our heads.

So to get back to the question at hand, let us reconsider the lighthouse. Only this time, neither of us is standing directly under it. Furthermore, neither of us knows for certain what the height of the lighthouse may be. Now the "side view" looks something like this:

Now each of us is an unknown distance from a lighthouse of unknown height.

Both of us can measure an altitude of the light.

Your altitude is greater (that is, closer to 90°) than mine because you’re closer.

Now we are missing all the data except the measurement of altitudes! Each of us has an altitude and it seems natural for us to compare our answers. This part is where someone had a great idea.

*The decision was made to define one nautical mile as that length needed to see a difference in altitude of one minute.*

**The Intercept**

Although the diagram is not to scale, please humor me and suppose my sextant reading were 38° 32.6’ and yours 38° 42.8’. Then you may simply take the difference between our measurements, which is 10.2’. Since one minute is one mile, we know immediately that you are precisely 10.2 miles closer to the lighthouse than me. That is to say, starting from my position, the intercept, denoted “a” in the diagram, is 10.2’ toward the lighthouse. To make it really work, the intercept must be toward the lighthouse on some definite azimuth, but we don’t need to choose one for this illustration.

Now you should be able to see for yourself how this goes. You don’t really need me in the picture. You could just as well say, "Let us decide on a ‘convenient spot’ for me to stand and call it my reference position”. I don’t really need to take a measurement, because once I know what time it is I’ll use arithmetic to compute what altitude and azimuth to the lighthouse I would measure from there. Since a computation comes entirely from mathematics, it doesn’t matter whether we can actually measure the azimuth or not. Then we’ll compare the computed altitude from the ‘convenient spot’ to what I actually observed on deck using my sextant.

There are 3 possible outcomes for the comparison:
1. The *observed altitude*, denoted Ho, is exactly the same as the *computed altitude*, denoted Hc. Here we must conclude that we are standing exactly upon the same equal altitude circle as our "convenient spot" (assumed or reference position) for which we did the calculation. Good guesses like this are rare.

2. The *observed altitude* Ho is less than the *computed altitude* Hc. Here we must conclude that we are standing farther away from the center of the circle computed for our assumed position.

3. The *observed altitude* Ho is greater than the *computed altitude* Hc. Here we must conclude that we are standing closer toward the center of the circle computed for our assumed position.

**Back to Captain Marq de St Hilaire**

Captain St Hilaire learned that the Nautical Almanac data could be used to locate the GP of a celestial body as normal, but after locating the GP, also was sufficient to allow him to choose virtually any position (latitude & longitude) and then be able to calculate the altitude an observer would measure of that particular celestial body, if the observer were actually located at that position.

Of course, he would not choose just any position he would choose a reference position close to where he believed he was located, such as his DR position. He would then calculate what the altitude of the celestial body would be from that location and then compare that calculated altitude to his actually observed altitude to learn if there was a difference. If the two altitudes, the observed and calculated, were exactly the same then he could conclude that the ship was indeed located at the reference position when the sight was taken. If the altitudes differed then he was not located at the reference position and his position was actually "off" by the difference in altitudes.

Here's an example:

An observed altitude (Ho) of 21° 23.7' results in a COP with a radius of 4116.3 nm. Co-Altitude = 90° - Altitude = 90° - 21° 23.7' = 68° 36.3' (68.605°) X 60 = 4116.3 nm.

Suppose we calculated an altitude (Hc) from a reference position that resulted in an Hc of 21° 21.6' with a resulting radius of 4118.4 nm.

If we compare the radii of the circles 4118.4 – 4116.3 we have a difference of 2.1 nm. Now here's the magical part that Captain St Hilaire discovered: Instead of determining Co-Altitudes (90 – Ho and 90 – Hc) and calculating and comparing the radii, just compare Ho to Hc. In our example Ho is 21° 23.7', and Hc is 21° 21.6' what is the difference between them? 2.1 nm the same as when we compared the radii!! So, we learn the difference without having to calculate Co-Altitude.

But, what does the 2.1 nm difference mean? It means that at the time of our observation of the celestial body with the sextant we were actually a distance of 2.1 nm from the reference position! So, at this point, we know a bit more but we'll have to determine the bearing to use to a plot point 2.1 nm different. We also need to determine if the 2.1 nm is in a direction closer to the GP or farther away.

By comparing Ho & Hc we can see that if Ho is greater than Hc we must have been 2.1 nm closer to the GP or if Hc is greater than Ho we must have been 2.1 nm farther away. (See the earlier narrative about altitude as we approach the GP).

Now, having two known locations on the globe, 1) the GP of the body and 2) the reference position, Captain St Hilaire learned he could mathematically calculate the azimuth from the reference position to the GP. This is the azimuth the ship must have been on at the moment he took the sextant reading!
Once the azimuth is calculated, we can now plot the reference position (L, Lo) on our active chart, and then measure and plot a position 2.1 nm TOWARD the GP (if Ho > Hc as in our example) or AWAY from the GP (if Ho < Hc) along the azimuth calculated as the azimuth from the reference position to the GP and thus we arrive at a plotted latitude & longitude as our Estimated Position (EP).

It is an Estimated Position because we have only a single sight. To be able to plot a fix we'd need to take a second sighting (see Figure 1507 above) of another celestial body (within 20 minutes of time) or if the celestial body is the sun, we can wait and sight the sun a second time 2 – 6 hours later and plot that LOP and advance the earlier LOP for a Running-Fix.

Does all this mean we can just forget about Co-Altitude? No! Our Law of Cosines formulas that we'll use to calculate Hc and the azimuth to the GP will use Co-Altitude, et al to arrive at the solutions we seek. But we need not bother with calculating and comparing the radii of the COPs, we'll just compare Ho and Hc to find the difference, which is called the intercept, and determine if the intercept is TOWARD or AWAY from the GP and plot that point along the calculated azimuth, hence the name “Intercept – Azimuth” method.

It's appropriate to call our preselected point a reference position, because it refers to the geographical area where we want to plot our LOP. This reference point can be anywhere but is usually a position chosen near the ship and, is typically our DR position but, does not have to be. Traditionally, in the jargon of Celestial Navigation, this point has been called the Assumed Position (AP) but that is misleading, we are not assuming we are located there. The AP simply says we want to locate a portion of the COP nearest this reference location. Don’t get confused, no assumptions are being made.

**Time**

The observation of a celestial body to determine position is dependent on the time of the observation. For practical purposes, the celestial bodies are fixed in the sky, but the earth is rotating at approximately 1000 mph! That suggests that every second, the celestial body’s location in the sky, relative to our position, is moving. (Actually, we are moving.) Thus, when we observe an altitude of a celestial body from Earth, its longitudinal position is changing every second! This renders the precision in the measurement of time associated with an altitude observation of a celestial body of utmost importance.

Beyond precision, there are the conventions of time measurement to be considered. For example, there are 24 time zones across the globe. These are typically established by political entities for various purposes, including ease of commerce and communication, i.e., to help our societies function more efficiently. Time, across a particular time zone, is constant, regardless of the relationship of the sun or celestial bodies. This is an issue for the navigator, given the precision that is required to track celestial bodies.

The solution to the challenge of time precision and specificity is the use of Universal Time (UT). The Nautical Almanac utilizes UT in pinpointing celestial bodies. Therefore, it is the responsibility of the student/navigator to learn how to convert local time to UT in order to extract the appropriate data from the Almanac.
**Time, a Further Discussion**

The Earth’s orbit around the Sun is a slightly elliptical one, with a mean distance from the Sun equal to 1 Astronomical Unit (AU = 80,795,193 nautical miles). This means that the Earth is sometimes a little closer to and sometimes a little farther from the Sun than 1 AU. When it is closer, the gravitational forces between Earth and Sun are greater and it is like going downhill where the Earth travels a little faster thru its orbital path. When it is farther away, the gravitational forces are less and it is like going uphill where the Earth travels a little slower.

Because the Earth’s orbit is not perfectly circular and its orbital velocity is not constant, the precise measurement of time is affected. We keep time by the movement of the Sun called *solar time*. For keeping time on our clocks, the movement of the Sun across the sky is averaged to become an imaginary *mean* Sun (establishing *mean time*) where every day is exactly 24 hours long and noon occurs at 1200 every day. However, the real or *apparent* Sun (establishing *apparent time*) is not constant and leads us to the time-related event that may be most familiar, the leap year, where we must add a day to our calendar every four years to keep our calendar time synchronized.

Navigators keep watch time like the rest of the world, using mean solar time however they also must learn about and use appropriate apparent time, the Sun’s *actual* movement. Throughout the year “noon”, when the Sun is directly over the observer’s meridian of longitude, actually varies from occurring 16 minutes early at 1144 watch time around early November to 14 minutes late at 1214 watch time around mid-February. This difference in time between *mean* time and *apparent* time is called the *Equation of Time* and is listed on the Daily pages of the Nautical Almanac. We typically ignore this difference for day-to-day living but to the mariner, it can mean the difference between a pleasant cruise and loss of the vessel or worse!

The Earth is a sphere\(^1\) with 360° of circumference and sunrise occurs every 24 hours. The Sun travels 360° every 24 hours. Therefore, the Sun must move across the sky at an average of 15° per hour (360° ÷ 24) or 1° every 4 minutes (60 ÷ 15). Mathematicians have determined that, at the Equator, 1° = 60 nautical miles. If a mariner’s time were incorrect by 16 minutes a position error of 4° or 240 nautical miles could result! Mariners must be able to determine apparent time as well as mean time.

In addition to mean and apparent time, the Earth’s circumference is divided into 24 one-hour (15°) time zones. See the figure below right. The time zones begin (or end) at the Greenwich Meridian (0° longitude), the “Z” or “Zulu” time zone, spanning 7½° east and west of the 0° meridian and progress both easterly and westerly from Greenwich every 15° (1 hour) to meet at the *International Date Line* at 180° longitude.

Midnight at Greenwich occurs when the Sun is over the 180° meridian (International Date Line). Noon at Greenwich occurs when the Sun is directly over the 0° meridian. The “world time” kept at Greenwich is called *Universal Time* (UT), formerly “*Greenwich Mean Time*”. The times of celestial body movements

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\(^1\) Actually an oblate spheroid; a sphere slightly flattened at the poles.
tabulated in the Nautical Almanac for each hour of each day are listed in UT. The mariner must also have the skill to convert local time (Zone Time (ZT)) of his/her time zone to UT in order to extract appropriate data from the almanac. Remember, the Sun rises in the east and sets in the west. The time on a clock in a time zone to the east is ahead of the time on a clock of another time zone to the west.

For example, if a mariner were located at 128° W longitude and zone time is 1045, what is UT? First, divide the longitude 128° by 15 to get 8.5333; and round to the nearest integer of 9 as the time zone. The mariner is 9 times zones west of Greenwich. Greenwich is 9 time zones east of the mariner and 9 hours ahead of the mariner’s ZT. Therefore, UT is: 1045 + 9 = 1945 UT at Greenwich. If the mariner were at 37° E longitude at ZT 1045, what is UT? 37 ÷ 15 = 2.4666 rounded to 2. Greenwich is two time zones west of the mariner therefore UT is 2 hours behind ZT, UT = 1045 – 2 = 0845 UT.

If a mariner’s position is in west longitude, one hour per time zone is added to local time to determine UT; if a mariner’s position is in east longitude, one hour per time zone is subtracted from local time to determine UT. Noon in any time zone occurs when the Sun is directly over the central meridian of the zone. Mariners keep Standard Time aboard ship; daylight time is ignored.

Time is longitude! The Earth rotates one revolution (360° of longitude) in one day. It therefore turns one degree of longitude in 1/360th of a day, or every four minutes. 24 hours per day X 60 minutes per hour = 1440 minutes ÷ 360° = 4 minutes per degree. To calculate your longitude, you simply need to determine the time difference between time at your location and time at the Greenwich (0°) Meridian.

Local Mean Time
Another time topic that a mariner’s knowledge requires is called Local Mean Time (LMT). LMT accounts for the difference in longitude between a mariner’s position and the central meridian of his/her time zone. We learned above that for each degree of longitude time differs by 4 minutes. Time zones are 15° wide with ½ the zone (7½° or 30 minutes) to the east and ½ (7½° or 30 minutes) to the west of the central meridian of the zone and noon for a time zone is when the Sun is directly over the central meridian of the zone.

Here’s an example to demonstrate LMT: A mariner is located at 133° W longitude. At what time will the Sun be over the mariner’s meridian (local noon at that position)? We know that noon for the time zone is when the Sun is over the central meridian of the time zone. Which meridian is the central meridian? Divide the longitude of 133°W by 15° per zone and the result is 8.8667 rounded to 9. The mariner’s time zone is 9 time zones west of Greenwich and 9 * 15 yields 135° W as the central meridian of the time zone. However, the mariner at 133°W is located 2° east of the central meridian of 135°W. The Sun will cross 133°W before it will cross 135°W. Therefore, Local Apparent Noon (LAN) at 133° W must occur earlier than 1200. How much earlier? We learned earlier that the Sun moves 1° every 4 minutes. We are 2° east of the zone meridian of 135° W so local noon must occur 8 minutes before noon at the central meridian or at 1152 ZT. Thus the LMT of noon at 133° W is 1152. Had the mariner waited until 1200 ZT to take a “noon sight for latitude”, he/she would have “missed” local noon by 8 minutes or 120 miles!

Now suppose the mariner were at 137°W. At what time would local noon occur? Did you get 1208? Remember, zone time is average (mean) time and we as a society agree that all clocks within the zone read 1200 when the Sun is over the central meridian of our time zone. However, this demonstration highlights the fact that Local Apparent Noon (LAN) occurs earlier for those east of the
central meridian and later for those west of the central meridian. The mariner must account for his/her difference in longitude from the central meridian when taking (noon) sights. \( DLo = Lo - ZM \) the difference in longitude (DLo) is our longitude minus the longitude of the central meridian (Zone Meridian (ZM)).

Consider the following example:

At what time must a mariner be ready to take a noon sight for latitude if the mariner is located at Lo 55° 25’W? The mariner must determine the LMT of noon. Solution: Determine the ZM of the time zone: Lo = 55 + (25/60) = 55.41667° ÷ 15 = 3.69444 rounded to 4. The mariner is 4 time zones west of Greenwich; central meridian (ZM) is 4 * 15 = 60°W. \( DLo = 55.41667° - 60° = -4.5833° \). The time difference is \(-4.5833° * 4 \text{ min per } ° = 18.3332 = -18 \text{ minutes 20 seconds}\). The LMT of noon = 12-00-00 – 18 m 20 s = 11-41-40. The mariner's longitude is 4.5833° east of the ZM so LMT of noon is 18 minutes and 20 seconds earlier than noon at the ZM. The mariner must be prepared to begin taking sights before 11-41-40.

**Use of the Nautical Almanac**

As stated earlier, the almanac is published yearly and the mariner must have possession of the almanac for the current year in order to accurately determine his/her position. For the following explanation, any edition of the almanac may be used. If one is not available, you can click on “online nautical almanac” to download a pdf version and use it to follow along. Note, however, the Increment and Corrections pages (explained below) and some other printed almanac data are not shown in the online version and the layout and values may differ slightly from the printed almanac.

The following page is an example that shows the left-hand and right-hand daily pages of the Nautical Almanac for April 9, 10, & 11 of 2004.

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2 [https://thenauticalalmanac.com/2017%20Nautical%20Almanac.pdf](https://thenauticalalmanac.com/2017%20Nautical%20Almanac.pdf)
These pages are called the “Daily pages”. Note that 3 days are covered across both the left and right-hand daily pages. The left-hand daily page tabulates Greenwich Hour Angle (GHA) for each whole hour UT of Aries (explained later), the GHA and Declination of the navigational planets, and alphabetically lists data for 57 navigational stars. GHA is the hourly longitudinal position of the celestial bodies measured westerly from the Greenwich Meridian through 360°. The bottom of the left-hand page also shows the time of local time zone central meridian passage of Aries and the planets, and the \( v \) and \( d \) values used to determine small corrections to be applied to your sight based on minutes and seconds of your sight time.

The right-hand daily page tabulates hourly UT data for the Sun and Moon and solar phenomena of Twilight times, Sunrise, Sunset, Moonrise, and Moonset at various latitudes. On the lower right of the right-hand page the Equation of Time (shaded if the apparent Sun is later than the mean Sun, clear otherwise), Time of the Sun’s meridian passage (noon at your local time zone’s central meridian), and Moon data are listed for each of the three days covered. This 3 day UT hourly tabulated format continues throughout the entire year of the almanac.

Mariners may take sextant sightings at any convenient time, not just on the hour. So, the almanac also contains a set of Increments and Corrections pages (pages ii to xxxi) (not available in the online almanac) near the back of the almanac. These pages list the incremental change in GHA for Sun & Planets, Aries, and Moon for each second of each minute within an hour. Also listed within each minute are the \( v \) or \( d \) corr which are small corrections to GHA (\( v \)) and declination (\( d \)) that may be required to be applied to your sextant sighting. The Increments and Corrections pages remain unchanged year to year and are the same regardless of the edition of the almanac you may have.

Twilight times are periods of incomplete darkness when it may be possible to view stars and planets and still be able to discern the horizon. The midpoint for optimal sights of the stars and planets is the beginning of Civil Twilight in the morning and the end of Civil Twilight in the evening.

The Nautical Almanac contains a plethora of other celestial related data and you can learn much by browsing but, that is another topic and will not cover here.

**Celestial Bodies and Their Geographical Position (GP)**

**The Celestial Sphere**

The mariner’s view of the cosmos for celestial navigation imagines a huge transparent celestial sphere, like a bubble, with the Earth at its center as shown in the figure at right. The celestial sphere is our star map where the fixed stars and moving planets are mapped on the inside of this sphere like a planetarium.

We realize that the stars and planets have a three-dimensional location in space, but for the purposes of navigation, we only need to know their direction and altitude in the sky. We are located on the Earth’s surface inside of the sphere looking out. The celestial sphere has an equatorial plane called the Celestial Equator, it is the Earth’s equator projected outward onto the sphere. The Earth’s
poles are also extended to the sphere as north and south celestial poles. The sphere does not spin. It is fixed in space while the Earth rotates inside it.

When we take a sextant sight of a celestial body to measure the altitude of the body, we imagine a line passing through our body, head to toe, extending to the celestial sphere. The spot directly over our head on the sphere is called the zenith and the spot directly below our feet is the nadir. This, along with the horizon, aids us in creating a triangle that we can use to determine other values.

The figure at right depicts a 2D graphical representation of a sextant sight. Note the Zenith to Horizon angle is 90° and the Observed Altitude measured plus the Co-Altitude span the entire 90°. We record the sextant observation data and the Co-Altitude tells us the distance from the GP of the body.

For example, if we measured an observed altitude of 50°, the Co-Altitude would be 90° - 50° = 40° and 40° at 60 nm per degree = 2400 nm distant from the Geographical Position (GP) of the body. Again, the Geographical Position of the body is a point on the Earth’s surface where a line connecting the Earths’ center to the celestial body passes through the Earth’s surface.

What is Aries, and How Does It Relate to Celestial Bodies?

The Seasons

Each year as the Earth revolves around the Sun we experience the four seasons. Each season begins at the time of either an equinox or a solstice. A solstice begins the Northern Hemisphere summer ~June 21st when the declination of the Sun’s GP is ~23.5° north of the equator at the Tropic of Cancer. Another solstice begins the Northern Hemisphere winter ~December 21st when the declination of the Sun’s GP is ~23.5° south of the equator at the Tropic of Capricorn. Equinoxes begin the Northern Hemisphere spring and fall when the declination of the Sun’s GP is at 0.0°on the equator. The Vernal Equinox occurs ~ March 21st and the Autumnal Equinox occurs ~ September 21st each year. Again, because the Earth’s actual movement is not constant, dates of solstices and equinoxes may vary between the 20th to the 22nd of the months indicated.

Return to page 30 and take another look at the image of the Celestial Sphere. Now imagine in your mind’s eye, the stars on the sphere as if you were viewing a planetarium and gazing at a sky full of stars. From Earth, the stars appear fixed in position on the sphere which is why you can see the same constellations year after year that move as the Earth rotates.

With the planetarium image in mind think of how you might identify the precise location of each visible star in the sky. Based on your learning so far you might think of measuring GHA and Declination as we did with the Sun and planets and you would be correct! Now think again, how many stars do you see? Imagine if you will, compiling an almanac with a column for each star recording the GHA &
Declination for each hour of each day for a year. How many columns would it have and how many pages long would it be? Could you conveniently carry it around?

The solution to this dilemma is straightforward. Because the stars seem fixed in the sky, we choose a single “start point” on the sphere and measure each star’s longitudinal position from that point and measure each star’s declination above or below the Celestial Equator normally. But what start point do we use? Enter Aries or also called the “First Point of Aries”!

Aries is the Vernal Equinox (March) when the Sun’s GP is on the equator and the Sun’s declination is 0.0°, projected onto the celestial sphere. It is used as the start (zero) point to measure each star’s longitudinal position on the sphere. The longitudinal position, from Aries to each star is called the star’s Sidereal Hour Angle (SHA). Each star’s declination north or south of the celestial equator has been measured and recorded normally. Declination of each star changes very slowly over time eliminating the need to record declination for each hour. If you select a star from the list on a left-hand page of the almanac for a specific date then look for that star’s declination in later pages you'll see the slow declination change for yourself.

Open your almanac to any date daily page and look on the right side of the left-hand page. Here you'll find an alphabetical listing of 57 stars chosen for navigation due to their distribution across the sky and their brightness making them visible to the naked eye. Note that each star listed has a SHA and declination recorded. Also, on the left side of the left-hand page is a column showing GHA Aries. There is no declination recorded because Aries is not a visible body and is always on the equator.

The GHA of a star is found using the formula  \( GHA\ Star = GHA\ Aries + SHA\ star \) (±360° as may be necessary). Here's how that is accomplished.

A sextant sighting is taken of the star Deneb on February 12, 2017, at 18-00-30 from a DR position of 47° 24.0’N 122° 20.1’W. What would be Deneb’s GHA? First, convert 18-00-30 12 Feb to UT. The time zone of Lo 122° 20.1’W becomes 122.335° / 15 = 8.1556 rounded to 8. The DR is west of Greenwich so 8 hours must be added to get UT. 18-00-30 becomes 26-00-30 12 Feb. There is no hour 26 so, subtract 24 hours and add 1 day; the UT is 02-00-30 13 Feb.

From the almanac, for 13 Feb extract the SHA Deneb of 49° 30.2’ and for hour 02, add GHA Aries of 173° 18.1’ then add to that 0° 07.5’ from the Aries column of the Increments and Corrections page for minute 0 second 30. The Total GHA becomes 222° 55.8’. Deneb’s longitudinal position at 02-00-30 on 13 Feb is 222° 55.8’ west of the Greenwich meridian at a declination (latitude) of 45° 20.5’ N. We have precisely located the GP of Deneb at the time of the sextant sight.

The Navigational Triangle: Possible Orientations

Now take a closer look at the navigational triangle to gain a better understanding. The navigational triangle can have different orientations based on whether the DR is in the Northern or Southern Hemisphere and whether the celestial body viewed is east or west of the DR, as shown in the figures below. The orientation will determine the correct method of determining the true azimuth (Zn) from the DR to the GP based on the result of the Law of Cosines formula for angle Z.
North Latitude

In North latitude, the azimuth $Z_n$ is determined by either $Z_n = 360° - Z$ or $Z_n = Z$ depending upon whether the GP is west of the observer when $LHA < 180°$ or east of the observer when $LHA > 180°$. The azimuth $Z$ is labeled $N \ XXX.X \ W$ when the latitude of the DR is north and the GP lies west of the DR or $N \ XXX.X \ E$ when the latitude of the DR is north and the GP lies east of the DR.

South Latitude

In South latitude, the azimuth $Z_n$ is determined by either $Z_n = 180° + Z$ or $Z_n = 180° - Z$ depending upon whether the GP is west of the observer when $LHA < 180°$ or east of the observer when $LHA > 180°$. The azimuth $Z$ is labeled $S \ XXX.X \ W$ when the DR latitude is south and the GP lies west of the DR or $S \ XXX.X \ E$ when the DR latitude is south and the GP lies east of the DR.

The Navigational Triangle: Solving for Unknowns - the Law of Cosines

A Sighting of the Sun at Meridian Passage or Transit – A Noon Sight for Latitude

When taking a sextant sighting, a mariner collects and records data about the sighting for later solving of the navigational triangle, called “Sight Reduction”, to obtain position data. The data
recorded must be organized and categorized properly so, the mariner typically uses a “Sight Reduction form”. You can find one for download to follow along here SR-96A³.

Sight Reduction

The Four Altitudes
Before delving into the details of sight reduction and to alleviate confusion, an explanation of the four “altitude names” used in sight reduction is warranted.

- Sextant Altitude (Hs) – The actual reading taken from the sextant when the sighting is measured.
- Apparent Altitude (Ha) – The sextant altitude corrected for the height of our eyes above sea level (Dip) and any instrument error (Instrument Correction (Ic)) inherent in the sextant and is then used as an entering argument to extract additional correction data from the almanac.
- Observed Altitude (Ho) – The apparent altitude additionally corrected for atmospheric refraction and other data extracted from page A2 (or A3) of the almanac for the body observed.
- Calculated Altitude (Hc) – The altitude of the celestial body calculated from the DR reference position.

Reducing a Sun Sight
Here’s how a sight is recorded and solved. Example #1:

On January 5, 2017, from a DR position of 47° 24.0’N 122° 20.1’ W at watch time 12-14-59 a sight is taken on the lower limb of the Sun and a sextant altitude (Hs) of 19° 55.1’ is measured. The height of eye above sea level is 15 feet; sextant Index Error (IE) is 01.5’ off the arc. There is no watch error.

Let us first dissect this example. The Sun is large and appears (when viewed using sextant Sun shades!) as a disk. To measure the altitude using a sextant, the operator must view the Sun and, using the sextant moveable arm (index arm), bring the reflected Sun down to the horizon such that either the bottom edge (Lower Limb) or top edge (Upper Limb) just touches on the visible horizon as shown at right.

At that moment, the operator calls “Mark” and an assistant records the watch time: seconds first, then minutes, then hours. The watch used may be fast or slow (determined beforehand) and if it is, watch error is also recorded. The observer then reads the sextant and records the altitude measured and the limb aligned (LL or UL).

When taking a sextant sight, the height of your eyes will be above sea level by your height plus the height of the platform upon which you are standing. The higher your eyes, the farther away the horizon appears. We must account for this “horizon error”, called “Dip of the Horizon” or just Dip for short. On the right-hand side of page A2 of the almanac is a Dip table that shows the correction required for various heights. Find the values that bracket your height and record the value shown. Or calculate it yourself using the formula: Dip correction = 0.97*√height in feet. For

³ https://www.pbps.org/docs/sr96a.pdf
example, if the height of your eye is 15 feet: the square root of 15 is 3.87; multiplied by 0.97 is 3.75 rounded to 3.8’. Dip correction is always negative and subtracts from your sextant measurement.

Also, sextants (particularly plastic sextants) may have “Index Error”. Index error is determined beforehand by setting the sextant to 0° 0.0’ and sighting the horizon then adjusting the micrometer drum such that both the viewed and reflected horizons align as shown above. The sextant reading is the index error. The reading may be above zero (on the arc) or below zero (off the arc (60’ – reading)). If it is “on the arc” that amount must be subtracted from the sextant reading, if it is “off the arc”, add that amount to the sextant reading.

Follow this link to learn more about sight corrections with graphical representations.

Now, back to the example: On January 5, 2017, from a DR position of 47° 24.0’N 122° 20.1’ W at watch time 12:14:59 a sight is taken on the lower limb of the Sun and a sextant altitude (Hs) of 19° 55.1’ is measured. The height of eye above sea level is 15 feet; sextant Index Error (IE) is 01.5’ off the arc. There is no watch error.

Step 1: The observed sight: Record the date, time, watch error, the DR, the name of the body observed, the height of eye above sea level, the sextant measurement (Hs), and Index Error. Next,
is completed. The figure above shows the “Observed Sight” data as recorded on the sight reduction form for Time, Sight Data, and Altitude. The purple cells show results of calculations.

Again, the process for determining the observed altitude (Ho) value is: Measure and record Hs using the sextant, correct Hs for IC & Dip to obtain Ha, extract the appropriate almanac correction based on Ha, and adding (if LL was sighted) or subtracting (if UL was sighted) the main correction to Ha to obtain the resulting Ho.

**Step 2: The calculated sight:** Called “Sight Reduction”. This is done by first selecting a reference position, latitude and longitude. Many celestial navigation texts refer to this location as an “Assumed Position” but that is a misnomer, we are not assuming we are located there. It is just a location in our vicinity that is used as a reference. Typically the DR position is used, however, any location within 30’ of our DR will work. The reference position along with data extracted from the almanac about the celestial body is used to construct a Navigational Triangle. We’ll not actually create the triangle the scientific calculator will do that for us as we solve the Law of Cosines formula. The formula will calculate the altitude of the celestial body that we would have measured if we were actually located at the reference position at the time of the sighting. Once we have calculated the altitude (Hc), we compare the altitude we observed (Ho) to the altitude calculated (Hc) to find the difference. The resulting difference tells us how far our location at the time of the sight was “offset” from the reference position but not the direction. Using the second Law of Cosines formula, the azimuth Z from the reference position to the GP of the body is calculated. The two Law of Cosines formulas are shown below.

\[
\sin H_c = \frac{\cos LHA \cdot \cos Lat \cdot \cos Dec}{H_s - (\sin Lat \cdot \sin Dec)} + \frac{(\sin Lat \cdot \sin Dec)}{H_s}
\]

\[
\cos Z = \frac{(\sin Dec - (sin Lat \cdot \sin H_c)) \cdot (\cos Lat \cdot \cos H_c)}{H_s}
\]

As can be seen in the 1\textsuperscript{st} formula, the LHA, latitude of the reference position, and Declination are used to find the value of Hc. In the 2\textsuperscript{nd} formula, declination, latitude, and Hc are used to solve for angle Z. To solve the “Calculated Sight” local time must be converted to UT, LHA determined from longitude and GHA data from the almanac, and declination data extracted from the almanac.

The following figures show a completed Sight Reduction form SR-96a for the example.
**Step 1: The observed sight:** We record the date, time, watch error, the DR, the name (and limb if Sun or Moon) of the body observed, the height of eye above sea level, the sextant measurement (Hs), and Index Error.

Next, we apply corrections:

- Sextant Altitude - Hs 19° 55.1’
- Index Correction – Ic +01.5 (Index Correction is the opposite sign of Index Error)
- Dip Correction –3.8’ (from the Dip table on page A2 of the almanac)

**Apparent Altitude – Ha** 19° 52.8’

Using Ha as the entering argument into App Alt column of page A2 of the almanac, find the App Alt entries that bracket our Ha and we extract the Main correction for a Lower Limb Sun sighting from the Sun table, Oct-Mar column as +13.6’

- Apparent Altitude – Ha 19° 52.8’
- Main correction +13.6

**Observed Altitude – Ho** 20° 06.4’

Ho is the altitude measured from our actual location. We’ll now compare Ho to what the altitude of the Sun would be from our DR position by calculating that altitude (Hc) using the latitude and longitude of our DR position as a reference along with data about the Sun, at the time of our sighting, extracted from the almanac.

**Step 2: The calculated sight:** First, our watch time of 12-14-59 must be converted to UT. Our DR longitude is 122° 20.1’ (122.335°) W. The Sun moves across the sky East to West at 15° per hour. 122.335 ÷ 15 = 8.15566 rounded to 8 hours west of Greenwich. Time at the Greenwich Meridian is therefore 8 hours later than our local time. UT = 12-14-59 + 8 = 20-14-59.

We open the almanac to 2017 JANUARY 4, 5, 6 (WED., THURS., FRI). From the right-hand daily page day 5 Sun column at whole hour 20 we extract and record the GHA of the Sun as 118° 35.0’ and the Dec of the Sun as 22° 30.8’ S and the d correction at the bottom of the Sun column as 0.3.

Next we turn to the **INCREMENT AND CORRECTIONS** page for minute 14 and follow down the column to second 59 and extract the change in GHA during 14m 59s as 3° 44.8’. Also on the 14m page from the **v or d Corr** column we find the **d** of 0.3 and extract the correction as 0.1’.

We now combine the whole hour GHA with the change in GHA for 14m 59s to get total GHA:

- 118° 35.0’
- +03° 44.8’
- 122° 19.8’ **Total GHA.**

We now combine our DR longitude (- if west, + if east) with our Total GHA to determine our Local Hour Angle (LHA).

<table>
<thead>
<tr>
<th>Total GHA</th>
<th>122° 19.8’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude</td>
<td>122° 20.1’ W (-)</td>
</tr>
<tr>
<td>LHA</td>
<td>359° 59.7’</td>
</tr>
</tbody>
</table>
LHA is the longitudinal angular difference between our DR (local) longitude and the longitude of the GP of the Sun. We see in this example that this difference is near zero, meaning the Sun’s GP is nearly on our DR meridian of longitude, making this Sun sight a “Noon Sight for Latitude”.

Now we turn to Declination. Earlier we extracted a dec of 22° 30.8’ S. We also extracted a d corr of 0.1’. Is the d corr to be added or subtracted? Return to the right-hand daily page for 2017 JANUARY 4, 5, 6 (WED., THURS., FRI) and find the dec for the next whole hour (21) of UT. Is the dec increasing further South or less South? It is 22° 30.5’ S or less South, so the d corr is subtracted.

<table>
<thead>
<tr>
<th>Dec</th>
<th>22° 30.8’ S</th>
</tr>
</thead>
<tbody>
<tr>
<td>d corr</td>
<td>-0.1</td>
</tr>
<tr>
<td>Total Dec</td>
<td>22° 30.7’ S</td>
</tr>
</tbody>
</table>

We now have enough information to solve for our calculated altitude Hc using the Law of Cosines formulas.

\[
\sin H_c = (\cos LHA \times \cos Lat \times \cos Dec) + (\sin Lat \times \sin Dec)
\]

\[
\cos Z = (\sin Dec - (\sin Lat \times \sin H_c)) + (\cos Lat \times \cos H_c)
\]

Convert the degrees and minutes of LHA, DR Latitude, and Declination to degrees to a precision of 5 decimal places. Note: If DR Latitude and Declination have the same name (both N or both S) we’ll enter dec as positive, otherwise negative.

<table>
<thead>
<tr>
<th>LHA</th>
<th>359° 59.7’ = 359.99500°</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR Latitude</td>
<td>47° 24.1’N = 47.40000°</td>
</tr>
<tr>
<td>Declination</td>
<td>22° 30.7’S = -22.51167°</td>
</tr>
</tbody>
</table>

Solving the first formula results in a sin Hc of 0.343468413. Using the sin-1 function of our calculator results in an Hc of 20.08833° or 20° 05.3’. We now compare the Ho to Hc:

<table>
<thead>
<tr>
<th>Ho</th>
<th>20° 06.4’ altitude actually measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hc</td>
<td>20° 05.3’ altitude calculated using DR as reference position</td>
</tr>
<tr>
<td>Intercept</td>
<td>01.1’</td>
</tr>
</tbody>
</table>

The intercept tells us that at the time of our sight we were 1.1 nautical miles from our DR position. Because Ho is greater than Hc, we must have been 1.1 nautical miles closer to the Sun’s GP than our DR position. What is the azimuth (bearing) to the Sun’s GP? We must solve formula #2.

Solving the second formula results in a cosine Z of -1. Using the cos-1 function on our calculator results in Z = 180°. The Sun’s GP true azimuth (Zn) is Zn = Z (in this example). Zn = 180°. So in this example the Sun’s azimuth (Zn) is 180° T from our DR position. The Sun’s GP is due south. See the table below for determining Zn (True Azimuth) from Z. Note also that, in this example, either DR Lat N formula works because the Sun’s GP is on our DR meridian of longitude.

<table>
<thead>
<tr>
<th>LHA</th>
<th>DR Lat</th>
<th>Zn</th>
<th>Zn = 360° - Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;180°</td>
<td>N</td>
<td>Zn = 180° + Z</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>Zn = 180° - Z</td>
<td></td>
</tr>
</tbody>
</table>
Note the true azimuth (DR to GP) Zn is 180° True. Compare the reference position latitude to the Sun’s declination and knowing that they are positioned on the same meridian an azimuth of 180° should make sense. The Sun is directly south of the reference position. Also, compare the values of Ho and Hc. In this case, Ho > Hc by 1.1’ and is labeled TOWARD. This means that at the time of the sight we were located 1.1’ (nm) closer to the GP than the reference position is located.

**Plot our Estimated Position**

Find our DR on our plotting sheet and plot a location 1.1nm from our DR on azimuth 180°T TOWARD (closer to (Ho > Hc)) the GP (due south in this example) and that plot becomes our estimated position!

**Reducing A Star Sight**

How do we solve a star sighting? Example #2:

*A sextant sighting is taken of the star Deneb on February 12, 2017, at 18-00-30 from a DR position of 47° 24.0’N 122° 20.1’W. Sextant altitude is 25° 57.5’; height of eye is 15 feet; IE is 01.5’ off the arc; there is no watch error. The SR-96a is shown below::*

Just as we did with the Sun sight, we record the date, time, watch error, the DR, the name (and limb if Sun or Moon) of the body observed, the height of eye above sea level, the sextant measurement (Hs), and Index Error.

Next, we apply corrections:

- Sextant Altitude - Hs = 25° 57.5’
- Index Correction – Ic +01.5 (Index Correction is the opposite sign of Index Error) = 25° 59.0’
- Dip Correction = -3.8’ (from the Dip table on page A2 of the almanac)
- Apparent Altitude – Ha = 25° 55.2’
Using Ha as the entering argument into App Alt column of page A2 of the almanac, find the App Alt entries that bracket our Ha and we extract the Main correction from the STARS AND PLANETS table as \(-2.0'\)

Apparent Altitude – Ha \(25°\ 55.2'\)
Main correction \(-2.0'\)
Observed Altitude – Ho \(25°\ 53.2'\)

Again, our watch time of 18-00-30 must be converted to UT. \(UT = 18-00-30 + 8 = 26-00-30 = 02-00-30\) the next day (13 Feb).

Determining the GHA of a star is a bit different. The GHA of a Star = GHA ARIES + SHA Star \((\text{GHA} \star = \text{GHA} \star \text{ries} + \text{SHA} \star)\). On the left-hand daily page for 13 February find the ARIES column and extract GHA at whole hour 02 on the 13th of 173° 18.1'. Also extract and record the SHA of Deneb from the STARS table of 49° 30.2' and Deneb’s declination of 45° 20.5' N.

Turn to the INCREMENT AND CORRECTIONS page for minute 0 and find second 30. Extract the change in GHA from the ARIES column of 0° 07.5'.

Now combine the GHA ARIES \((\gamma)\) at 0200 with the changes of GHA for 30 seconds and the SHA of Deneb. Then combine with SHA Deneb \((\pm360\ if\ necessary)\)

\[
\text{173° 18.1'}\ \text{hour 0200} \\
+00° 07.5' \text{ change in 30 sec} \\
\text{173° 25.6'} \text{ GHA} \gamma. \\
\text{049° 30.2'} \text{ SHA} \star \\
\text{222° 55.8'} \text{ Total GHA} \star
\]

We now combine our DR longitude \((-\ if\ west, +\ if\ east)\) with our Total GHA to determine our Local Hour Angle (LHA).
We record the declination of Deneb as a total dec of 45° 20.5’ N (there is no v or d corr for stars).

Convert the degrees and minutes of LHA, DR Latitude, and Declination to degrees to a precision of 5 decimal places. Note: If DR Latitude and Declination have the same name (both N or both S) we’ll enter dec as positive, otherwise negative.

\[
\begin{align*}
\text{LHA} & \quad 100° 35.7’ = 100.59500° \\
\text{DR Latitude} & \quad 47° 24.1’N = 47.40000° \\
\text{Declination} & \quad 45° 20.5’N = 45.34167°
\end{align*}
\]

Solving the Law of Cosines formulas results in a \( \sin H_c \) of 0.43611776 thus an \( H_c \) of 25° 51.4’ and a \( \cos Z \) of 0.640109699 thus a \( Z \) of 50.2° and a \( Z_n \) of 310° T. Deneb’s GP is west of our longitude.

Comparing \( H_o \) to \( H_c \):

\[
\begin{align*}
H_o & \quad 25° 53.2’ \text{ altitude actually measured} \\
H_c & \quad 25° 51.4’ \text{ altitude calculated using DR as reference position} \\
\text{Intercept} & \quad 01.8’
\end{align*}
\]

Plot our Estimated Position

Find our DR on our plotting sheet and plot a location 1.8 nm from our DR on azimuth 310°T TOWARD (closer to) the GP (Ho > Hc) and that plot becomes our estimated position!

The Planets

Unlike the fixed stars, the planets move among the stars throughout the year. This makes locating planets that may be visible for sights a bit more challenging. The chart on page 9 of the almanac shows the LOCAL MEAN TIME OF MERIDIAN PASSAGE of the planets throughout the year. The narrative on page 8 is helpful in deciphering the chart. To interpret the chart, for the Northern Hemisphere (turn the chart upside down for the Southern Hemisphere and directions are opposite), is to set the (2017) almanac in front of you in portrait orientation (have the sun shaded area going directly away from you; you are facing south); East is to the left and West is to the right. Now imagine it is noon on the 20th of July. From the chart, you can see that Venus is to the right (west) of the Sun.
so it sets before the Sun and therefore will not be available at evening twilight but may be available at morning twilight as it rises before the Sun.

Jupiter is East of the Sun so it sets well after the Sun as it crosses your meridian at ~1700, five hours (75°) after the Sun so will be available at evening twilight in the SW.

Saturn is also East of the Sun (and East of Jupiter) and crosses your meridian ~2145 so it may be available at evening twilight to the SE as it is well after the Sun.

If you could roll this diagram into a tube along the Sun axis, and rotate it westerly, the planet actions become even easier to imagine.

Note that Mercury is shown on the chart but is too close to the Sun most of the time to be of use to the navigator. It is included in the chart so the navigator can determine when it is in conjunction with other planets (where their tracks cross) and can eliminate the possibility of confusion of the sight between Mercury and the planet of interest.

Another way to help locate planets is to use the SHA and LMT of meridian passage of each planet that is shown on the lower right of the left-hand daily page of the almanac. Planets whose meridian passage time is more than an hour away from the Sun’s meridian passage time are potentially visible during morning or evening twilight. If a planet’s meridian passage is earlier than the Sun’s, the planet will set ahead of the Sun and will not be visible after sunset but may be available before sunrise. If a planet’s meridian passage is near midnight it rises near sunset and will be visible all night.

The altitude of the planet (any celestial body) is a function of (the planet’s) declination and your latitude. Altitude at meridian passage (Transit) can be predicted with the following formula:

\[
\text{Altitude at MT} = \text{Co-Latitude} \pm \text{Declination}
\]

Where Co-Latitude is 90° - your latitude and declination are added if declination and co-latitude have the same name (both north or both south), subtracted otherwise. Co-Latitude is north if the body’s GP is south of the observer. Knowing this altitude can assist you in estimating when the planet will rise or set and whether it may be available for morning or evening twilight.

Here’s an example of a planet sight. Example #3

On February 15, 2017, from a DR position of 47° 24.0’N 122° 20.1’W at 18-05-00, a sight of Mars is taken. The height of the eye is 15 feet; IE is 01.5’ off the arc; Hs is 34° 41.5’; there is no watch error.

Determine Ha and Ho.

Apply IE & Dip to Hs to determine Ha = 34° 39.2’.

Extract the main correction from page A2 of the almanac STARS AND PLANETS table (-1.4) and Additional Corr for Mars (+0.1) for a total correction of -1.3 and a Ho of 34° 37.9.

Convert our time of 18-05-00 to UT. UT = 18-05-00 + 8 = 26-05-00 or 02-05-00 next day (16 Feb).

From the left-hand daily page for 16 Feb MARS column extract GHA Mars at 0200 of 163° 19.5’ and Dec Mars of 05° 22.6’ N with v of 0.8 and d of 0.7.

From INCREMENTS AND CORRECTIONS page for minute 5 second 00 record the change in GHA of 01° 15’ and v or d corr of 0.1’ for v and 0.1 for d.

Determine Total GHA of Mars by adding GHA 0200, change in GHA, and v corr to get 164° 34.6’
Combine with DR longitude to determine LHA of 42° 14.4’ W.

Combine declination with d corr for Total Dec of 05° 22.7’ N

Solving the Law of Cosines formulas results in a sin Hc of 0.567891728 thus an Hc of 34° 36.2’ and a cos Z of -0.58212297 thus a Z of 125.6° and a Zn of 234° T. Mars’ GP is west of our longitude.

Comparing Ho to Hc:

Ho 34° 37.9’ altitude actually measured
Hc 34° 36.2’ altitude calculated using DR as reference position

**Intercept** 01.7’

Here’s the completed SR-96a:

<table>
<thead>
<tr>
<th><strong>Time</strong></th>
<th><strong>Sight Data</strong></th>
<th><strong>Altitude</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Date: 02 15 2017</td>
<td>Sight No: 3</td>
<td>Hi of eye: 15.0 feet</td>
</tr>
<tr>
<td>WT: 18 5 0</td>
<td>Body: Mars</td>
<td>Degrees: Min</td>
</tr>
<tr>
<td>WE Seconds</td>
<td>DR L: 47 24.0 N</td>
<td>Hs: 34 41.5</td>
</tr>
<tr>
<td>ZT: 18 05 0</td>
<td>DR Lo: -122 -20.1 W</td>
<td></td>
</tr>
<tr>
<td>ZD + 8 W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UT: 02 05 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G Day/Mo: 16 02 2017</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ALMANAC - LHA**

<table>
<thead>
<tr>
<th>SHA</th>
<th>Degrees</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 hr</td>
<td>103</td>
<td>19.5</td>
</tr>
<tr>
<td>5 min</td>
<td>01</td>
<td>15.0</td>
</tr>
<tr>
<td>V (+)</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Total GHA</td>
<td>164</td>
<td>34.8</td>
</tr>
<tr>
<td>DR Lo (-)</td>
<td>-122</td>
<td>-20.1 W</td>
</tr>
<tr>
<td>LHA</td>
<td>042</td>
<td>14.5 W</td>
</tr>
</tbody>
</table>

**ALMANAC - Dec**

<table>
<thead>
<tr>
<th>Dec 2_Hr</th>
<th>d (+)</th>
<th>d corr (+)</th>
<th>Dec N</th>
</tr>
</thead>
<tbody>
<tr>
<td>05.0</td>
<td>22.6 N</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>22.7 N</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To increase your proficiency, print a blank SR-96a and, using an almanac and following the details shown in previous examples, try solving this sight. It’s fine if you need to refer back often, just give it a try. If you do not have a current almanac, you can use the one you have for practice, keeping in mind the extracted data will not match precisely.

Here’s the Mars Sight Reduction solution:
Plot our Estimated Position

Find our DR on our plotting sheet and plot a location 1.7 nm from our DR on azimuth 234°T TOWARD (closer to) the GP (Ho > Hc) and that plot becomes our estimated position!

You may also find it interesting to note the mathematical relationship of sines to cosines used by the formulas. We learned in the Navigational Triangle that we are solving for the Co-H side, yet the 1st formula results in the Sine of Hc. In the solution above, Hc is 34° 36.2' or 34.60333°. The Co-Altitude (Co-H) is then 90° – 34.60333° = 55.39667°.

Use your calculator cosine function to find the cos(55.39667°) and store the result. Now use your calculator's sin⁻¹ (ASin) function and using the cosine stored, find the resultant angle, sin⁻¹(cosine 55.39667). Did you get (Hc) 34.60333°? The sine of an angle is equal to the cosine of the angle’s complement! Sine 34.60333° = cosine 55.39667°. The formula really has solved for the cosine of Co-H but used the sin⁻¹ function to find its Hc complement.

Reducing a Moon Sight

Altitude corrections for the moon are more complex than those for the other bodies. The correction table for the moon (located in the back of the Nautical Almanac, pages xxxiv and xxxv) has two parts. The upper part is entered with ha, as for the sun, star, and planet tables. Another factor, called horizontal parallax (HP), is needed to enter the bottom part of the table. Because the moon’s position relative to the earth changes rapidly, horizontal parallax values are tabulated for hourly intervals in the daily pages of the Almanac.

On March 04, 2017, from a DR position of 47° 24.0’N 122° 20.1’W at 18-20-30, a sight of the Moon is taken. The height of the eye is 15 feet; IE is 01.5’ off the arc; Hs is 58° 02.5’; there is no watch error.
Determine Ha and Ho.

Apply IE & Dip to Hs to determine Ha = 58° 00.2′.

Convert our time of 18-20-30 to UT. UT = 18-20-30 + 8 = 26-20-30 or 02-20-30 next day (5 Mar).

Record GHA Moon from the right-hand daily page for 5 Mar at 0200 along with a v of 7.0, the declination of the Moon as 16° 40.4′ N with a d of 5.5 and HP of 59.3.

Using Ha as the entering argument, extract the main correction from page xxxv of the almanac ALTITUDE CORRECTION TABLES 35° - 90° - MOON table (+40.6) and Additional Corr (bottom ½ of table) for Lower Limb (L) HP 59.3 (+6.3) for a total correction of 46.9 and a Ho of 58° 47.1′.

- Apparent altitude (58°00.2′) is used to enter the table, so find the column that includes your value of ha this is the column headed 55°-59°. These numbers refer to degrees of apparent altitude. Follow that column down until you come to 58°.
- Minutes of apparent altitude are listed at the sides of the table. You are looking for 00.2′. Follow across the table from 00′ and you find the value of +40.6′. The next entry, for 10′, has a value of +40.5′. Since 00.2′ is closer to 00′ than 10′, the main altitude correction value for 00.2′ is +40.6′. (You must interpolate to find the main altitude correction). The main altitude correction for the moon is always positive.
- In addition to this main correction, moon sights also require an additional correction. For the additional moon correction, use the bottom part of the table. The columns on each side are headed HP, and the inner columns are headed L and U. The L is for lower-limb sights, the U for upper-limb sights. In the same column as before (55°-59°), drop to the lower part of the table.
- Go down the L side of the column until you bracket the HP value of 59.3′ you got from the daily page. You will find an HP of 59.1′, which corresponds to a value of 6.1′. Below that is an HP of 59.4′, which corresponds to a value of 6.4′. Interpolate between these bracketed values to obtain the additional moon correction for HP = 59.3′, which is +6.3′. These additional corrections are always positive.
From INCREMENTS AND CORRECTIONS page for minute 20 second 30 record the change in GHA of 04° 53.5’ and v or d corr of +2.4’ for v and +1.8’ for d.

Determine Total GHA of the Moon by adding GHA 0200, change in GHA, and v corr to get 129° 21.4’

Combine Total GHA with DR longitude to determine LHA of 07° 01.3’ W.

Combine declination with d corr for Total Dec of 16° 42.2 N’

Solving the Law of Cosines formulas results in a sin Hc of 0.855034533 thus an Hc of 58° 45.8’ and a cos Z of -0.974370065 thus a Z of 167° and a Zn of 193° T. The Moon’s GP is west of our longitude.

Comparing Ho to Hc:

<table>
<thead>
<tr>
<th>Intercept and Azimuth by the Law of Cosines for Sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHA</td>
</tr>
<tr>
<td>Lat N</td>
</tr>
<tr>
<td>Dec N</td>
</tr>
</tbody>
</table>

\[
Hc = \text{ASIN}\left[\frac{\text{COS}(LHA) \times \text{COS}(Lat) \times \text{COS}(Dec)) + (\text{SIN}(Lat) \times \text{SIN}(Dec))}{\text{SIN}(Hc)}\right]
\]

\[
Z = \text{ACOS}\left[\frac{\text{SIN}(Dec) - (\text{SIN}(Lat) \times \text{SIN}(Hc)) - (\text{COS}(Lat) \times \text{COS}(Hc))}{\text{COS}(Zn)}\right]
\]

| Intercept-> | 1.3 nm | TOWARD |
| Z----------> | 167.0 | = 166 57° N 167° W |
| Zn----------> | 193° True |

Plot our Estimated Position

Find our DR on our plotting sheet and plot a location 1.3 nm from our DR on azimuth 193°T TOWARD (closer to) the GP (Ho > Hc) and that plot becomes our estimated position!
Two Body Fix

Here we'll use the previous Moon Sighting and take a sighting of Venus to establish a Two-Body Fix.

On March 04, 2017, from a DR position of 47° 24.0'N 122° 20.1'W at 18-32-15, a sight of Venus is taken. The height of the eye is 15 feet; IE is 01.5' off the arc; Hs is 22° 30.0; there is no watch error.

Determine Ha and Ho.

Apply IE & Dip to Hs to determine Ha = 22° 27.7'.

Extract the main correction from page A2 of the almanac STARS AND PLANETS table (-2.3) and Additional Corr for Venus Feb 19 – Mar 6 (+0.4) for a total correction of -1.9 and a Ho of 22° 25.8.

Convert our time of 18-32-15 to UT. UT = 18-32-15 + 8 = 26-32-15 or 02-32-15 next day (05 Mar).

From the left-hand daily page for day 05 Venus column extract GHA Venus at 0200 of 183° 43.8’ and Dec Venus of 11° 40.9’ N with v of 2.8 and d of 0.3 at bottom of page.

From INCREMENTS AND CORRECTIONS page for minute 32 second 15 record the change in GHA of 08° 03.8’ and v or d corr of 1.5’ for v and 0.2 for d.

Determine Total GHA of Venus by adding GHA 0200, change in GHA, and v corr to get 191° 49.1’

Combine with DR longitude to determine LHA of 69° 29.0’ W.

Combine declination with d corr for Total Dec of 11° 41.1’ N

Solving the Law of Cosines formulas results in a sin Hc of 0.381393081 thus an Hc of 22° 25.2’ and a cos Z of -0.125333234 thus a Z of 97.2° and a Zn of 263° T. Venus’ GP is west of our longitude.

Comparing Ho to Hc:

<table>
<thead>
<tr>
<th>Ho</th>
<th>22° 25.8’ altitude actually measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hc</td>
<td>22° 25.2’ altitude calculated using DR as reference position</td>
</tr>
<tr>
<td>Intercept</td>
<td>00.6’</td>
</tr>
</tbody>
</table>

Plot our Estimated Position

After plotting our previous Moon sighting EP, find our DR on our plotting sheet and plot a location 0.6 nm from our DR on azimuth 263°T TOWARD (closer to) the GP (Ho > Hc) and that plot becomes our estimated position. Extend the Moon LOP and the Venus LOP, where they intersect becomes our 1832 fix of position and we read the Lat/Long from our plotting sheet and record in the Log.

The SR 96 and solution follows:
### Intercept and Azimuth by the Law of Cosines for Sides

LHA \( \rightarrow \) LHA  
\[ \text{LHA} = 69.290 \quad \rightarrow \quad \text{LHA} = 69.48333° \]

Lat N \( \rightarrow \) Lat ( + )  
\[ \text{Lat} = 47.240 \quad \rightarrow \quad \text{Lat} = 47.40000° \]

Dec N \( \rightarrow \) Dec ( + )  
\[ \text{Dec} = 11.411 \quad \rightarrow \quad \text{Dec} = 11.68500° \]

\[ \text{Hc} = \arcsin[(\cos(LHA) \times \cos(Lat) \times \cos(Dec)) + (\sin(Lat) \times \sin(Dec))] \]

\[ Z = \arccos[(\sin(Dec) - (\sin(Lat) \times \sin(Hc))) \div (\cos(Lat) \times \cos(Hc))] \]

\[ \text{Hc} = 22.42027 = 22° 25.2' \]

\[ \text{Ho} = 22.43000 = 22° 25.8' \]

**Intercept->** 0.6 nm TOWARD  
**Z------->** 97.2 = 097° 10’ N 97.2° W  
**Zn------->** 263° True
Test Your Skills
Access the online 2019 Almanac to use for the skills test.

1. You’ve taken a sighting of the lower limb of the sun and your Ho is 41° 37.2'. What is your distance from the sun’s GP?

2. You’ve taken a sighting of the lower limb of the Moon and your Ho is 31° 17.7'. What is your distance from the moon’s GP?

3. You are located 3333 miles from the sun’s GP. What is the sun’s altitude?

4. You are located 1847 miles from a star’s GP. What is the star’s altitude?

5. A sextant sighting is taken of the star Sirius on March 10, 2019, at 18-40-15 PST from a DR position of 47° 24.0’N 122° 20.1’W. Sextant altitude is 24° 29.2'; height of eye is 15 feet; IE is 01.5' off the arc; there is no watch error. What is Ho?

6. A sextant sighting is taken of the planet Mars on March 10, 2019, at 18-48-10 PST from a DR position of 47° 24.0’N 122° 20.1’W. Sextant altitude is 43° 10.2'; height of eye is 15 feet; IE is 01.5' on the arc; there is no watch error. What is the intercept, Z, and Zn?

7. A sextant sighting is taken of the Sun LL on May 16, 2019, at 14-36-41 PST from a DR position of 47° 24.0’N 122° 20.1’W. Sextant altitude is 48° 13.7'; height of eye is 15 feet; IE is 01.5' on the arc; there is no watch error. What is the intercept, Z, and Zn?

8. What will the sun’s altitude be on the summer solstice June 21st, 2019 at Meridian Transit (LAN) from latitude 47° 24.0’N?
Test Your Skills Answers

1. Your distance from the GP is 2902.8 nm.
   \[90° - 41° 37.2' = 48° 22.8' (48.38°) \times 60\]

2. Your distance from the GP is 3522.3 nm.
   \[90° - 31° 17.7' = 58° 42.3' (58.705°) \times 60\]

3. The sun’s altitude is 34° 27.0’
   \[3333 \div 60 = 55.55° 90° - 55.55° = 34.45° = 34° 27.0’\]

4. The star’s altitude is 59° 13.0’
   \[1847 \div 60 = 30.78333° 90° - 30.78333° = 59° 13.0’\]

5. Ho is 24° 24.6’
   \[Hs = 24° 29.2’ + IC of 1.5 \text{ minus Dip 3.8 \text{ minus main corr 2.0}}\]

6. The intercept is 3.7 nm Away (Ho < Hc) azimuth \(Z = 114.2°\) thus \(Zn\) 246° T

<table>
<thead>
<tr>
<th>Time</th>
<th>Sight Data</th>
<th>Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>YYYY MM DD</td>
<td>Sight No.</td>
</tr>
<tr>
<td>WE S</td>
<td>HR MM SS</td>
<td>Body Mars</td>
</tr>
<tr>
<td>ZT</td>
<td>N</td>
<td>DR L 47 24.9</td>
</tr>
<tr>
<td>ZD +</td>
<td>W</td>
<td>DR L -122 -20.1</td>
</tr>
<tr>
<td>UT</td>
<td>02 48 10</td>
<td>G Day/Mo 11 03 2019</td>
</tr>
</tbody>
</table>

**ALMANAC - LHA**

<table>
<thead>
<tr>
<th>SHA Mars</th>
<th>Dec 2 Hr</th>
<th>Degrees Min</th>
<th>Main 0 010</th>
</tr>
</thead>
<tbody>
<tr>
<td>164 34.2</td>
<td>17.0 30.6</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>

**ALMANAC - Dec**

<table>
<thead>
<tr>
<th>SHA Mars</th>
<th>Dec N</th>
<th>Degrees Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 92.6</td>
<td>17 31.9</td>
<td>N</td>
</tr>
</tbody>
</table>

| Intercept-> | 3.7 nm AWAY |
| Z-----------> | 114.2° = 114 9’ N 114.2° W |
| Zn----------> | 246° True |
7. The intercept is 0.6 nm Toward (Ho > Hc) azimuth Z = 119.5° thus Zn 240° T

<table>
<thead>
<tr>
<th>Time</th>
<th>Sight Data</th>
<th>Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>Sight No</td>
<td>Ht of eye</td>
</tr>
<tr>
<td>MM</td>
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<td>H</td>
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<td>MM</td>
<td>M</td>
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<tr>
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<td></td>
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<tr>
<td>WT</td>
<td></td>
<td>Body</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>

| ZH  | DR L      | DR Lo    |          |     |
|     | Degrees   | Min      |         |     |
|     |           |          |         |     |

<table>
<thead>
<tr>
<th>WE Seconds</th>
<th>DST Y/N</th>
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</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

| ZD  | W       |          |         |
|     | Degrees | Min      |         |
|     |         |          |         |

| G Day/Mo |          |         |         |
|----------|----------|---------|

**ALMANAC - LHA**

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<tbody>
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<td></td>
<td></td>
</tr>
<tr>
<td>22 hr</td>
<td>150</td>
<td>54.1</td>
</tr>
<tr>
<td>36 m 41 s</td>
<td>09</td>
<td>10.3</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total GHA</th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>159</td>
<td></td>
<td>04.4</td>
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</table>

<table>
<thead>
<tr>
<th>DR Lo (-)</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>-122</td>
<td>-20.1</td>
</tr>
</tbody>
</table>

| LHA | 44.3 | W |

**ALMANAC - Dec**

<table>
<thead>
<tr>
<th>Dec 22 Hr</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.0</td>
<td>11.6</td>
</tr>
</tbody>
</table>

| d (+)     | 0.6 |
| dec (+)   | 0.4 |


<table>
<thead>
<tr>
<th>Dec N</th>
<th>N</th>
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</thead>
<tbody>
<tr>
<td>15</td>
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**HP C**

<table>
<thead>
<tr>
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</thead>
</table>

<table>
<thead>
<tr>
<th>Total</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ho</th>
<th>48</th>
</tr>
</thead>
</table>

| 23.5 |

**Intercept->** 0.6 nm TOWARD

**Z—->** 119.5° = 119° 30′ N 119.5° W

**Zn—->** 240° True

8. **Altitude at Meridian Transit = Co-Latitude ± Declination** 90° - 47.4° = 42.6° + 23.4° = 66°.
Advantages of Law of Cosines Method

Using the Law of Cosines Method of Sight Reduction requires that you have aboard only a current Nautical Almanac and a scientific calculator along with your standard plotting materials and tools.

Two other common methods of Sight Reduction require the use of a current Nautical Almanac along with either Pub 229 SIGHT REDUCTION TABLES FOR MARINE NAVIGATION (six volumes at ~$25.00 each) or Pub 249 SIGHT REDUCTION TABLES FOR AIR NAVIGATION (three volumes at ~$25.00 each). These publications each contain thousands of pre-calculated whole degree solutions of the Navigational Triangle and require slight manipulations of your DR latitude and longitude to achieve whole degrees of DEC, LAT, or LHA.

Difficulties

Perhaps the most difficult aspect of learning Celestial Navigation and Sight Reduction is remembering how to extract the myriad corrections and tabular data from the almanac for sightings of the Sun, Moon, Planets, and stars. It becomes easier only with practice.
Bibliography:
The American Practical Navigator by Nathaniel Bowditch
Celestial Navigation in the GPS Age by John Karl
Celestial Navigation in a Nutshell by Hewitt Schlereth
Dutton's Navigation & Piloting by Elbert S. Maloney
Marine Navigation Celestial and Electronic by Richard R. Hobbs
The Primer of Navigation by George W. Mixter
The USPS® Junior Navigation and Navigation student manuals past (pre 2006) and present editions.

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