Altitude – Intercept Method, a Description

By Ron Davidson, SN

Perhaps the most difficult and intimidating topic in learning Celestial Navigation for many, is understanding the trigonometry involved in the process of Sight Reduction to find position.

My objective herein is to explain some of the confusing and archaic terminology of Celestial Navigation and provide an easy understanding of what a sextant does and how the angles measured are used to determine position.

In the USPS Junior Navigation and Navigation courses we're taught how to find our position by what is termed the *Altitude-Intercept Method* of Sight Reduction. This article describes the essential concepts of *Altitude* and *Intercept* for understanding; it does *not* cover the mechanics of how to reduce a sight for position, that skill can be easily learned elsewhere.

Angles of celestial bodies above the horizon, measured using a sextant, are termed "*altitudes*" and the difference between two altitudes, once converted to nautical miles, is termed the "*intercept*". We'll see how the entire process is accomplished as we continue.

The *altitude-intercept* method involves observations with a sextant, but there is more work to be done since a sextant cannot, by itself, "yield your position." In this method, we must have data from both the sextant and also a very accurate watch. (Today's quartz and digital watches are very accurate). Taking a sextant observation is entirely worthless unless we know precisely when that observation was made. And even then, at least two such observations on two different objects are required to yield your position, known as a *fix*. The best a single observation can yield is an "*estimated position*".

Well just what does a sextant tell you?

A sextant is a very sophisticated instrument for measuring angles. Most of us are familiar with plastic protractors used for measuring angles in high school geometry or trig. These have one division for each degree, and most of us would agree that the one degree divisions are pretty small. Now consider taking each of those degrees and dividing it into 60 equal parts. Each of these parts we call one minute. A marine sextant can easily measure angles down to the minute.

If we then divide each of these into 60 parts, we have created 3600 arc seconds in only one degree. That is pretty fine division!

A marine sextant subdivides minutes, but not quite as finely as one second. Instead, it takes each minute of arc and divides it into fifths (but marked in tenths) as 0.0', 0.2', 0.4', 0.6', and 0.8'

Here's another complication which is a little confusing to the beginner; namely, mixing sexagesimal notation (that is, relating by sixtieths - degrees-minutes-seconds) with decimal notation. So what a Navigator gets from a sextant is a measurement of an angle, which will be reported in degrees, minutes and tenths of a minute. The data look something like: 27° 24.0' or 18° 04.2'.

Where do these angles come from?

They are angles measured between the sea horizon and objects in the sky. Remember, in celestial navigation, "angles" are referred to as "altitudes".

Do the altitudes then tell us our position?

Not directly, but they are a critical part of the whole process. The altitude at which a body appears in the sky is related to three conditions:

- The location of the body in space.
- The exact time of the observation.
- The position of the observer on the Earth.

Do you now get a hint of where this is going? This is like a high school algebra problem where you are given enough information to allow you to solve for that one bit you don't know. From the viewpoint of the navigator, here is where you get the data you need:

- The location of celestial bodies in space is compiled into the Nautical Almanac.
- The time is measured. “Old salts” used a chronometer, but today the typical digital quartz watch is more accurate than the very best chronometers of only two generations ago.
- The altitude of a star, a planet, the Moon or the Sun is measured using a sextant.

Now 3 things of 4 are known, and the final item of interest can be solved for; namely, the position of the observer on the Earth. That is the theory of celestial navigation. If it were a perfect world, this would be all we need; but there will be many complications to the complete solution of the altitude-intercept problem, so be patient.

Determine time!

This is the basis of our notions of "noon" and "time zones", and the reason why our "year" has the length it does. By the end of this article, I hope you will understand clearly how the motion of celestial bodies is tied to time. In fact, the motion of the Sun, actually defines our concept of time.

What is all this intercept nonsense about?

It is going to be a lengthy explanation, so bear with me...It will be worth the effort.

To begin, first notice that the spot directly over your head is always directly over your head. That spot is termed your *Zenith*.

As incongruous as it may seem this is one statement that is actually very important to your understanding of how the altitude-intercept method works.

Please also imagine that you are standing on a flat, level surface. If you now stand one arm of a carpenter's square between your feet pointing out toward the horizon, the other arm will point at that spot directly over your head, your zenith.

Now if to go just one step further... Imagine that the surface you’re standing on goes out as far as the eye can see. In other words, the surface extends all the way to your horizon.

If you are following this line of thought, it should be very clear now that the spot directly over your head is precisely 90° from the horizon, the carpenter's square is telling you so. And since the spot directly over your head is always directly over your head, it surely must be 90° from the horizon no matter where on Earth you may be standing.

This spot directly over your head is so special and important in celestial navigation that we give it a name: the *zenith*, which is denoted as Z.

But where does the intercept come in? We'll be there in a few minutes!

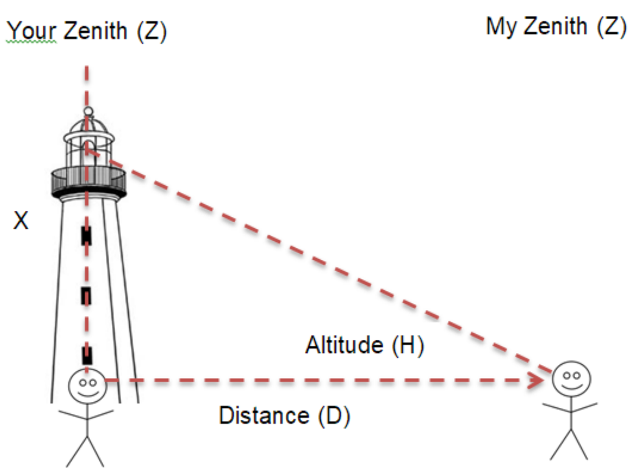
Let's pick out a very special zenith. Suppose that you are standing at the bottom of a lighthouse. Now there is a very bright light directly over your head that can be seen for miles around. I could ask you, "What is the altitude of the lighthouse light?" Now, knowing what you do about a zenith and an altitude, you would answer to me without even a measurement: "Why 90°, of course; since the lighthouse light is directly over my head at my zenith". And your answer would be exactly correct.

Now comes a truly interesting and crucial point in navigation. Your answer is perfectly sound, but unless I happen to be standing right with you there in the lighthouse, I will disagree with you as to the altitude of the light.

How so?

Because the spot directly over my head is always directly over my head! And it is a different spot than yours. Since my zenith is still a zenith, it is precisely 90° 00.0' from my horizon. Therefore, if I am not standing in the lighthouse with you, I will say that the altitude of the lighthouse light is less than 90°. If you doubt me, stand directly under a light fixture in your house. You will see that the angle between the floor and the light (your zenith) is 90°. Now take 3 steps backward and look at the light again. You will see that the angle between the floor, through you to the light is now less than 90°.

Fortunately for celestial navigation, there is a mathematical relationship that will tell me just what the altitude of the light will be. It depends on how far I am from the lighthouse and how tall the lighthouse is. See the diagram below.



In this diagram, you are in the lighthouse and I'm somewhere outside.

The lighthouse is X feet tall.

I am standing some D feet away from the lighthouse.

The altitude of the light is 90° for you, and some altitude, H, for me.

Now going back to the right triangles of high school trigonometry, we remember that the tangent of an angle is equal to the length of the "opposite side" divided by the length of the "adjacent side", written

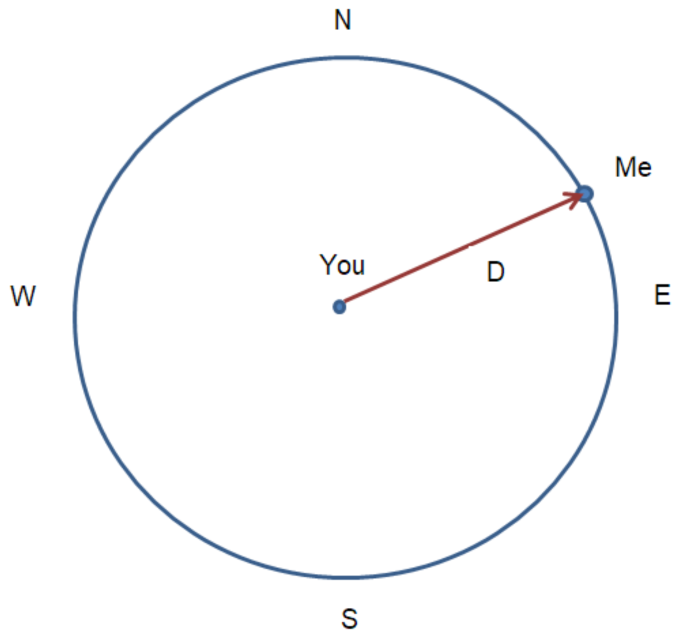
$$\tan H = (X / D)$$

Of course, the situation envisioned is that I have my handy sextant to measure altitude H, I just so happen to know the height of the lighthouse H, and I can then whip out my calculator to solve for distance between you and me, namely D.

Will you ever get down to the intercept? Patience!! It is closer than you think.

The intercept comes in because of two complications, one based in theory and one based in technology.

The previous diagram shows a "side view" of the lighthouse problem. The theoretical trouble is just this: With the information at hand, I cannot tell which "side" I am looking at. That is to say, I can tell the distance I may be from you, but not the direction. Am I D feet North of the lighthouse? D feet South? East? West? or somewhere in between. Here's an "overhead view" of all the possibilities.



In this diagram, we still see you in the lighthouse and me somewhere outside.

The Four Cardinal Directions around the horizon are shown.

I am standing some D feet away from the lighthouse.

Everywhere I may stand along the circle circumference the sextant will yield the same altitude of the light.

This very important phenomenon is called the *Circle of Equal Altitude*.

Circle of Equal Altitude

It also brings up something crucially important for developing the idea of an intercept.

Consider this point: if I were to stand off of the circle of equal altitude, my sextant reading would change. If I stand closer to you, the altitude will increase. It must because my zenith is getting closer to your zenith if I move inside the circle. If I take a step or two back, the altitude must decrease because my zenith is moving farther away.

Walking along a circle of equal altitude is something else you can try in your own room with that ceiling-mounted light again, and I strongly encouraged you to do so to affirm the point.

If we need direction, why don't we just use a compass and measure our bearing to the lighthouse? Then we would know everything we need.

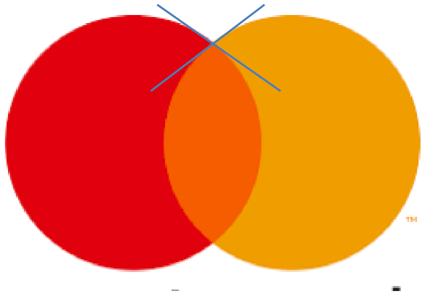
Sounds good! And it is routinely done in coastal navigation, where angular precision is less important because you are sighting a stationary landmark relatively nearby.

When our "lighthouses" become celestial bodies, we run into a technological problem.

Remember what was said about the precision of a sextant? A sextant easily measures down to fractions of a minute. Well, a compass doesn't. No one has yet devised a compass that gives better than fractions of a degree, so compass measurements are over 60 times less accurate than sextant measurements. This means that their relative uncertainty is large to begin with. Additionally, in normal practice the distance to our celestial "lighthouse" will be hundreds or even thousands of miles, so we would be taking a large uncertainty and multiplying it by a very large number, making it even more uncertain.

As if that weren't enough, when it comes time to plot our fix, we have no drafting instruments that will divide angles more finely than perhaps half a degree. All these large uncertainties add up quickly, and even though we don't seek "scientific precision" in our work, this level of uncertainty is just unacceptable.

This explains why we must measure at least two altitudes on two different bodies for a fix. A two-body celestial fix is the intersection of two circles of equal altitude. If you could draw the circles out completely on a globe, they would resemble the MasterCard symbol, as shown:



I say only resemble because in the circles won't be of equal size, but the graphic makes the point.

Our position is where the circles intersect. However, two circles taken from celestial sights normally cross each other twice, but the intersect positions are hundreds or even thousands of miles apart, so it is obvious which one is the position of our vessel. Since each altitude is known quite precisely, the intersection fix is far more precise than trying to take an altitude and a bearing, or azimuth, as it is called, of a single body.

The trouble appears when we go to actually plot our fix on the plotting sheet. The circles are huge and our plotting sheet too small. So, we approximate a small portion of the circle of equal altitude by a straight line, called a “line of position” as shown in the figure above. This is more practical than trying to draw a huge circle, but we still must know where the center of the circle lies. That is to say, we need to know what the azimuth to our "lighthouse" would be if we could measure it.

Are we there yet? The end is in sight!

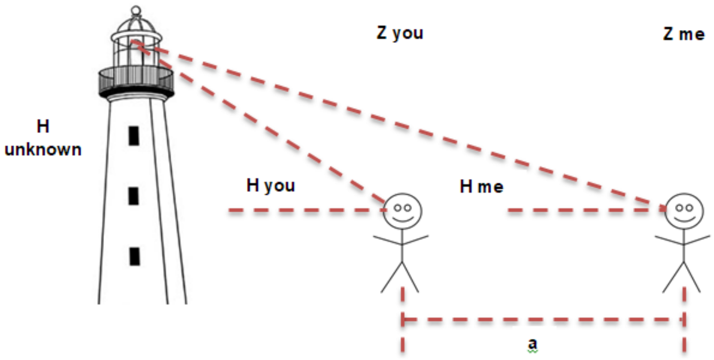
The final answer lies in some very real complications to the lighthouse pictures. If you haven't guessed by now, the lighthouses of which we are speaking are the Sun, Moon, planets, and stars. They have a huge advantage over man-made lighthouses in that they can be seen over entire hemispheres rather than just the local coastal region. But using celestial bodies leads to certain complications:

As viewed from the Earth, celestial bodies are moving in a very complex manner. With a few exceptions, stars rise, climb higher in the sky for a few hours, then descend toward the horizon and set. They appear again the next evening, but about 1° West of where they were the previous night at the same time. After a season or two, the whole sky looks different as once-familiar stars get lost in the glare of day for a few months, only to re-appear after a year has passed, right back where we first noticed them. This means that even in those extraordinarily rare instances that a bright star is in our zenith, it won't stay there long.

We also can't tell how "tall" these “lighthouses” are. Remember that our trigonometry solution depended on knowing the height of the lighthouse.

The Earth is round, not flat; and the sky also appears to be a big round dome over our heads.

So to get back to the question at hand, let us reconsider the lighthouse. Only this time, neither of us is standing directly under it. Furthermore, neither of us knows for certain what the height of the lighthouse may be. Now the "side view" looks something like this:



Now each of us is an unknown distance from a lighthouse of unknown height.

Both of us can measure an altitude of the light.

Your altitude is greater (that is, closer to 90°) than mine.

Now we are missing *all* the data except the measurement of altitudes! Each of us has an altitude and it seems natural for us to compare our answers. This part is where someone had a great idea.

> *The decision was made to define one nautical mile as that length needed to see a difference in altitude of one minute.*

At last, we come to the intercept!!

Although the diagram is not to scale, please humor me and suppose my sextant reading were 38° 32.6' and yours 38° 42.8'. Then you may simply take the difference between our measurements, which is 10.2'. Since one minute is one mile, we know immediately that you are precisely 10.2 miles closer to the lighthouse than me. That is to say, starting from my position, the *intercept*, denoted "a" in the diagram, is 10.2' toward. To make it really work, the intercept must be toward the lighthouse on some definite azimuth, but we don't need to choose one for this illustration.

Is that all there is to it?

Yes! Now you should be able to see for yourself how this goes. You don't really need me in the picture. You could just as well say, "Let us decide on a 'convenient spot' for me to stand and call it my assumed position". I don't really need to take a measurement, because once I know what time it is I'll use mathematics to compute what altitude and azimuth to the lighthouse I would see from there. Since a computation comes entirely from mathematics, it doesn't matter whether we can actually measure the azimuth or not. Then we'll compare the *computed* altitude from the 'convenient spot' to what I *actually observed* on deck using my sextant.

There are 3 possible outcomes for the comparison:

1. The *observed altitude*, denoted Ho, is exactly the same as the *computed altitude*, denoted Hc. Here we must conclude that we are standing exactly upon the same equal altitude circle as our "convenient spot" (assumed position) for which we did the calculation. Good guesses like this are rare.
2. The *observed altitude* Ho is less than the *computed altitude* Hc. Here we must conclude that we are standing farther away from the center of the circle computed for our assumed position.
3. The *observed altitude* Ho is more than the *computed altitude* Hc. Here we must conclude that we are standing closer toward the center of the circle computed for our assumed position.

We use the convention of writing Ho for *observed altitude* and Hc for *computed altitude*.

Summary of Plotting a Celestial Fix

We plot an assumed position[1] (typically our DR[2]), which is our 'convenient spot' that will have a Latitude and Longitude in our vicinity, on the plotting sheet. And, after extracting appropriate data from the Nautical Almanac about the body observed, we compute the altitude angle (Hc) that a sextant would measure from that Lat/Long. We then compare the computed altitude (Hc) to the observed altitude we actually measured (Ho) with our sextant.

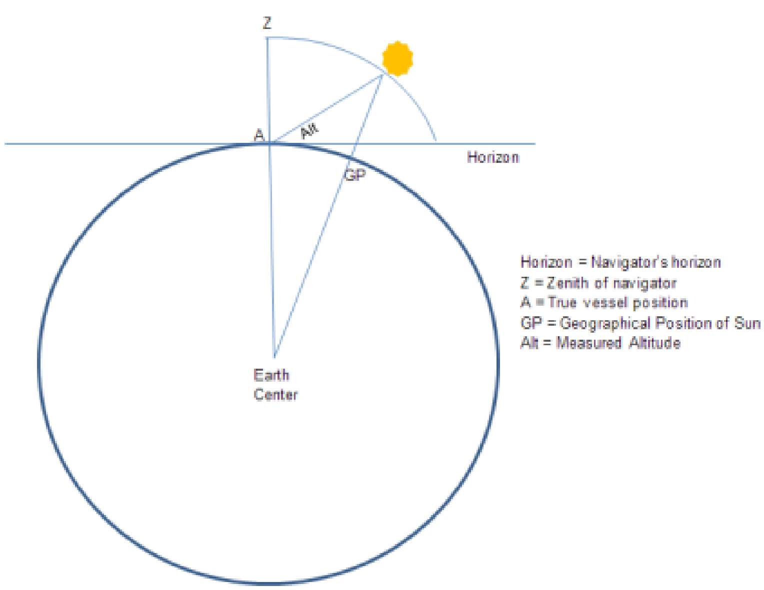
We take our computed angle (Hc) as a radius of a circle of equal altitude and our computed azimuth to the "lighthouse" for that assumed position. We take our observed angle (Ho) as a radius of a second circle of equal altitude and compare the results. The comparison results in our *intercept*.

We measure from our 'convenient spot' along the computed azimuth the number of miles specified by the intercept, either toward or away from the "lighthouse", following the rules:

Ho > Hc = a is toward or Ho < Hc = a is away. Where "a" is the intercept.

We plot our assumed position (usually our DR). We then draw a line perpendicular to the computed azimuth, through the point to which our intercept led us. This line is the "line of position" and represents a small segment of a circle of equal altitude as we actually observed it. This process is repeated for as many bodies as the Navigator cares to use, and the intersection of the lines of position is the *fix* location (Lat/Long) and can be read from the plotting sheet as our position.

We cannot physically measure the distance from the vessel to the Geographical Position (GP) of the celestial body (the center of our circle of equal altitude) but we can measure the altitude of the Sun at the true position and from that we can calculate the "zenith distance" as can be explained with the aid of the diagram below.



This diagram shows the 90° angle between Z and the horizon. Using our sextant, we measure the altitude of the Sun (Ho (Alt in the diagram)). We can now see the distance between our position and the GP is 90° minus Altitude. This is called the Zenith Distance (ZD) and is the radius of our Circle of Equal Altitude from the GP.

Now, if we calculate what the altitude (Hc) would have been at the Assumed Position (the DR) at the time that the altitude was measured at the true position, we are then able to compare the two altitudes and calculate the difference between them to see how far "off" we are from the DR position.

The difference between the two altitudes ($Ho - Hc = ZD_{Ho} - ZD_{Hc}$) is the *intercept*.

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[1] Don't get confused, the term is a misnomer. We are not assuming we are located there. We are just choosing a convenient Latitude and Longitude in our vicinity to use as a reference location for computing the altitude a sextant would read if the observation were taken from that location.

[2] Many student navigators think of the DR as imprecise and uncertain. It is not! The DR is a precise latitude and longitude that can be plotted. What is uncertain is our location; we may be at the DR or we may not. That's what we're trying to determine by following the navigation axiom "keep a DR and periodically verify the DR by other means".